Module 1: Relationships Between Quantities and Reasoning with Equations and Their Graphs (40 days)

Topic A: Graphing Stories – Introduction to Functions

By the end of eighth grade, students have learned to solve linear equations in one variable and have applied graphical and algebraic methods to analyze and solve systems of linear equations in two variables. Now, students analyze and explain precisely the process of solving an equation. Students, through reasoning, develop fluency writing, interpreting, and translating between various forms of linear equations and inequalities, and make conjectures about the form that a linear equation might take in a solution to a problem. They reason abstractly and quantitatively by choosing and interpreting units in the context of creating equations in two variables to represent relationships between quantities. They master the solution of linear equations and apply related solution techniques and the properties of exponents to the creation and solution of simple exponential equations.

In this unit, students solidify their previous work with functional relationships as they begin to formalize the concept of a mathematical function. This unit provides an opportunity for students to reinforce their understanding of the various representations of a functional relationship—words, concrete elements, numbers, graphs, and algebraic expressions. Students review the distinction between independent and dependent variables in a functional relationship and connect those to the domain and range of a function. The standards listed here will be revisited multiple times throughout the course, as students encounter new function families.

Big Idea:
- Units and quantities define the parameters of a given situation and are used to solve problems.
- The different parts of expressions, equations and inequalities can represent certain values in the context of a situation and help determine a solution process.
- Relationships between quantities can be represented symbolically, numerically, graphically, and verbally in the exploration of real world situations.

Essential Questions:
- When is it advantageous to represent relationships between quantities symbolically? numerically? graphically?
- What is the relationship between physical measurements and representations on a graph?
- How are appropriate quantities from a situation (a “graphing story”) defined?
- How is the scale and origin for a graph chosen and interpreted?

Vocabulary
- Piecewise-linear function, intersection point

Assessments
- Galileo: Algebra I Module 1 Foundational Skills Assessment; Live Binders/Galileo: Topic A Assessment

<table>
<thead>
<tr>
<th>Standard</th>
<th>Common Core Standards</th>
<th>Explanations &amp; Examples</th>
<th>Resources</th>
</tr>
</thead>
</table>
| N.Q.A.1    | A. Reason qualitatively and units to solve problems                                   | Maya and Earl live at opposite ends of the hallway in their apartment building. Their doors are 50 feet apart. They each start at their door and walk at a steady pace towards each other and stop when they meet. Suppose that:  
  - Maya walks at a constant rate of 3 feet every second.  
  - Earl walks at a constant rate of 4 feet every second. | Eureka Math:  
  Module 1 Lesson 1  
  Module 1 Lesson 2  
  Module 1 Lesson 3  
  Module 1 Lesson 4 |
| **N.Q.A.2** | **A. Reason qualitatively and units to solve problems**  
Define appropriate quantities for the purpose of descriptive modeling. | **N.Q.2** Determine and interpret appropriate quantities when using descriptive modeling.  
This standard will be assessed in Algebra I by ensuring that some modeling tasks (involving Algebra I content or securely held content from grades 6-8) require the student to create a quantity of interest in the situation being described.  
*Darryl lives on the third floor of his apartment building. His bike is locked up outside on the ground floor. At 3:00 p.m., he leaves to go run errands, but as he is walking down the stairs, he realizes he forgot his wallet. He goes back up the stairs to get it and then leaves again. As he tries to unlock his bike, he realizes that he forgot his keys. One last time, he goes back up the stairs to get his keys. He then unlocks his bike, and he is on his way at 3:10 p.m.*  
Sketch a graph that depicts Darryl's change in elevation over time.  
*The graph students produce should appear as follows:* | **Other**  
MAP – *interpreting distance-time graphs*  
Gizmos – *Distance-Time Graphs*  
**Eureka Math:**  
Module 1 Lesson 1  
Module 1 Lesson 2  
Module 1 Lesson 3  
Module 1 Lesson 4  
**Other:**  
MAP – *interpreting distance-time graphs*  
Gizmos – *Distance-Time Graphs*  
|
A ramp is made in the shape of a right triangle using the dimensions described in the picture below. The ramp length is 110 feet from the top of the ramp to the bottom, and the horizontal width of the ramp is 92.25 feet.

A ball is released at the top of the ramp and takes 1.6 seconds to roll from the top of the ramp to the bottom. Find each answer below to the nearest 0.1 feet/sec.

a. Find the average speed of the ball over the 1.6 seconds.
   \[ \frac{110}{1.6} \text{ ft./sec. or } 68.75 \text{ ft./sec.} \]

b. Find the average rate of horizontal change of the ball over the 1.6 seconds.
   \[ \frac{92.25}{1.6} \text{ ft./sec. or } 57.65 \text{ ft./sec.} \]

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**N.Q.A.3**

**A. Reason qualitatively and units to solve problems**

Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

Jason is collecting data on the rate of water usage in the tallest skyscraper in the world during a typical day. The skyscraper contains both apartments and businesses. The electronic water meter for the building displays the total amount of water used in liters. At noon, Jason looks at the water meter and notes that the digit in the ones place on the water meter display changes too rapidly to read the digit and that the digit in the tens place changes every second or so.

a. Estimate the total number of liters used in the building during one 24-hour day. Take into account the time of day when he made his observation. (Hint: Will water be used at the same rate at 2:00 a.m. as at noon?). Explain how you arrived at your estimate.

Since water is probably only used from about 5:00 am to 11:00 pm, I did not multiply by 24 hours, but by 18 hours instead.

Eureka Math:
- Module 1 Lesson 1
- Module 1 Lesson 2
- Module 1 Lesson 3
- Module 1 Lesson 4

Other:
- MAP – interpreting distance-time graphs
- Gizmos – Distance-Time Graphs
### A.CED.A.2

**A. Create equations that describe numbers or relationships**

Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

*Example*

Duke starts at the base of a ramp and walks up it at a constant rate. His elevation increases by three feet every second. Just as Duke starts walking up the ramp, Shirley starts at the top of the same 25 foot high ramp and begins walking down the ramp at a constant rate. Her elevation decreases two feet every second.

Write down the equation of the line that represents Duke’s motion as he moves up the ramp and the equation of the line that represents Shirley’s motion as she moves down. Show that the coordinates of the point you found in question above satisfy both equations.

If $y$ represents elevation in feet and $t$ represents time in seconds, then Duke’s elevation satisfies $y = 3t$ and Shirley’s $y = 25 - 2t$. The lines intersect at $(5,)$, and this point does indeed lie on both lines.

Duke: $15 = 3(5)$

Shirley: $15 = 25 - 2(5)$

Maya and Earl live at opposite ends of the hallway in their apartment building. Their doors are 50 feet apart. They each start at their door and walk at a steady pace towards each other and stop when they meet.

Suppose that:
- Maya walks at a constant rate of 3 feet every second.
- Earl walks at a constant rate of 4 feet every second.

Suppose that Maya walks at a constant rate of 3 feet every second and Earl walks at a constant rate of 4 feet every second starting from 50 feet away. Create equations for each person’s distance from Maya’s door and determine exactly when they meet in the hallway. How far are they from Maya’s door at this time?

**Maya’s Equation:** $y = 3t$.

**Earl’s Equation:** $y = 50 - 4t$.

Solving the equation $3t = 50 - 4t$, gives the solution: $t = \frac{7}{7}$. The two meet at exactly this time at a distance of $(\frac{7}{7}) = 2\frac{3}{7}$ feet from Maya’s door.

### MP.1

**Make sense of problems and persevere in solving them.**

Students are presented with problems that require them to try special cases and simpler forms of the original problem in order to gain insight into the problem.

### MP.2

**Reason abstractly and quantitatively.**

Students analyze graphs of non-constant rate measurements and reason from the shape of the graphs to infer what quantities are being displayed and consider possible units to represent those quantities.
| MP.3 | **Construct viable arguments and critique the reasoning of others.** | Students reason about water usage at the school; they construct arguments based on finding intersection points graphically and algebraically. | **Eureka Math:**  
Module 1 Lesson 1  
Module 1 Lesson 4  
Module 1 Lesson 5 |
|---|---|---|---|
| MP.4 | **Model with mathematics.** | Students have numerous opportunities in this module to solve problems arising in everyday life, society, and the workplace from modeling bacteria growth to understanding the federal progressive income tax system. | **Eureka Math:**  
Module 1 Lesson 2  
Module 1 Lesson 3 |
| MP.6 | **Attend to precision.** | Students formalize descriptions of what they learned before (variables, solution sets, numerical expressions, algebraic expressions, etc.) as they build equivalent expressions and solve equations. Students analyze solution sets of equations to determine processes (like squaring both sides of an equation) that might lead to a solution set that differs from that of the original equation. | **Eureka Math:**  
Module 1 Lesson 1  
Module 1 Lesson 3 |
Module 1: Relationships Between Quantities and Reasoning with Equations and Their Graphs (40 days)

Topic B: The Structure of Expressions

Students develop a precise understanding of what it means for expressions to be algebraically equivalent. By exploring geometric representations of the distributive, associative, and commutative properties for positive whole numbers and variable expressions assumed to represent positive whole numbers, students confirm their understanding of these properties and expand them to apply to all real numbers. Students use the properties to generate equivalent expressions and formalize that two algebraic expressions are equivalent if we can convert one expression into the other by repeatedly applying the commutative, associative and distributive properties, and the properties of rational exponents to components of the first expression.

Students learn to relate polynomials to integers written in base \(x\), rather than our traditional base of 10. The analogies between the system of integers and the system of polynomials continue as they learn to add, subtract, and multiply polynomials and to find that the polynomials for a system that is closed under those operations (e.g., a polynomial added to, subtracted from, or multiplied by another polynomial) always produces another polynomial.

| Big Idea: | • The different parts of expressions, equations and inequalities can represent certain values in the context of a situation and help determine a solution process.  
• Relationships between quantities can be represented symbolically, numerically, graphically, and verbally in the exploration of real world situations.  
• Rules of arithmetic and algebra can be used together with notions of equivalence to transform equations and inequalities.  
• Equivalent forms of an expression can be found, dependent on how the expression is used.  
• The Commutative and Associative Properties represent key beliefs about the arithmetic of real numbers. These properties can be applied to algebraic expressions using variables that represent real numbers.  
• Two algebraic expressions are equivalent if we can convert one expression into the other by repeatedly applying the Commutative, Associative, and Distributive Properties and the properties of rational exponents to components of the first expression. |
|---|---|
| Essential Questions: | • Why are the commutative, associative, and distributive properties so important in mathematics?  
• How are polynomials analogous to integers?  
• If you add two polynomials together, is the result sure to be another polynomial? The difference of two polynomials?  
• Is the product of two polynomials sure to be another polynomial?  
• Is a polynomial squared sure to be another polynomial (other integer powers)? |
| Vocabulary | numerical symbol, variable symbol, numerical expression, algebraic expression, equivalent numerical expressions, equivalent algebraic expressions, polynomial expression, monomial, degree of a monomial, degree of a polynomial, polynomial, leading term, leading coefficient, constant term, standard form |
| Assessments | Live Binders/Galileo: Topic B Assessment |

7/2/2014
A. Interpret the structure of expressions

Use the structure of an expression to identify ways to rewrite it. For example, see \( x^2 - y^2 \) as \((x^2)^2 - (y^2)^2\), thus recognizing it as a difference of squares that can be factored as \((x^2 - y^2)(x^2 + y^2)\).

Simplify expressions including combining like terms, using the distributive property and other operations with polynomials.

Tasks limited to numerical and polynomial expressions in one variable.

Recognize \(53^2 - 47^2\) as a difference of squares and see an opportunity to rewrite it in the easier-to-evaluate form \((53 + 47)(53 - 47)\). See an opportunity to rewrite \(a^2 + 9a + 14\) as \((a + 7)(a + 2)\).

Challenge the students to come up with more than one way to create the number 21:

<table>
<thead>
<tr>
<th>Value of Expression</th>
<th>Expression (using 1, 2, 3, 4, (+), and (-))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
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<tr>
<td>3</td>
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<td>9</td>
<td>9</td>
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<tr>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>11</td>
<td>4x + 3</td>
</tr>
<tr>
<td>12</td>
<td>4x</td>
</tr>
<tr>
<td>13</td>
<td>4x - 3</td>
</tr>
<tr>
<td>14</td>
<td>4x + 2</td>
</tr>
<tr>
<td>15</td>
<td>4x + 3</td>
</tr>
</tbody>
</table>

\[110 = 10 \cdot 10 + 10 = (1 + 2 + 3 + 4) \cdot (1 + 2 + 3 + 4) + (1 + 2 + 3 + 4)\]

Draw pictures to represent expressions \((x + y + 3) \times (y + 1)\).
Write equivalent expressions by applying the distributive property.

\[(x + y + 3)(y + 1) = xy + x + y^2 + 4y + 3\]

Rewriting expressions using associative and commutative properties:

The process of making use of the distributive property “backwards” is factoring.
Ex. The expression \(10x^2 + 6x^3\) is the result of applying the distributive property to the expression \(2x^2(5x^2 + 3x^3)\) or to \(x(10x + 6x^2)\); even to \(1(10x^2 + 6x^3)!\) \(\text{YOU ARE ARE NOT TEACHING THE FACTORING STEPS!}\)

\[
\frac{16x^2}{x^3} = 16x^5
\]

\[
(2x)^4(2x)^3 = 128x^7
\]

\[
(9z^{-2})(3z^{-1})^{-3} = \frac{z}{3}
\]
### A.APR.A.1

**A. Perform arithmetic operations on polynomials**

Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

| a. | 417 + 231 = 6 hundreds + 1 tens + 7 ones + 2 hundreds + 3 tens + 1 ones - 6 hundreds + 4 tens + 8 ones. |
| b. | (4x^2 + x + 7) + (2x^2 + 3x + 1). |
| c. | 6x^2 + 4x + 8 |
| d. | (x^3 - x^2 + 8) - (x^3 + 5x^2 + 4x - 7). |
| e. | 2x^3 - 6x^2 - 4x + 15 |
| f. | 3(x^2 + 8x) - 2(x^2 + 12). |
| g. | x^3 + 74x - 24 |
| h. | (5 - r - r^2) + (9t + r^2) |
| i. | 8t + 5 |
| j. | (3p + 1) + 6(p - 8) - (p + 2) |
| k. | 8p - 49 |

| (3 + x)^2 |
| z^2 + 6z + 9 |
| (5 + q)^3 |
| q^3 + 15q^2 + 75q + 125 |
| (u - 1)(u^5 + u^4 + u^3 + u^2 + u + 1) |
| u^6 - 1 |

### MP.7

**Look for and make use of structure.**

Students reason with and about collections of equivalent expressions to see how all the expressions in the collection are linked together through the properties of operations. They

**Eureka Math:**
- Module 1 Lesson 8
- Module 1 Lesson 9

**Big Ideas:**
- pp. 357-364, 365-370, 371-376
<table>
<thead>
<tr>
<th><strong>MP.8</strong></th>
<th><strong>Look for and express regularity in repeated reasoning.</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>They have opportunities to pay close attention to calculations involving the properties of operations, properties of equality, and properties of inequality as they find equivalent expressions and solve equations, noting common ways to solve different types of equations.</td>
</tr>
</tbody>
</table>

Eureka Math: Module 1 Lesson 7


## HS Algebra I Semester 1

### Module 1: Relationships Between Quantities and Reasoning with Equations and Their Graphs (40 days)

#### Topic C: Solving Equations and Inequalities

Students have written and solved linear equations and inequalities in their previous mathematics courses. The work of this unit should be on bringing students to mastery of this area of their mathematical study. This unit leverages the connection between equations and functions and explores how different representations of a function lead to techniques to solve linear equations, including tables, graphs, concrete models, algebraic operations, and "undoing" (reasoning backwards). This unit provides opportunities for students to continue to practice their ability to create and graph equations in two variables, as described in A-CED.A.2 and A-REI.D.10.

The Common Core Learning Standards rightfully downplay the notion of equivalent equations and instead place a heavy emphasis on students studying the solution sets to equations. First, students formalize descriptions of what they learned before (true/false equations, solution sets, identities, properties of equality, etc.) and learn how to explain the steps of solving equations to construct viable arguments to justify their solution methods. They then learn methods for solving inequalities, again by focusing on ways to preserve the (now infinite) solution sets. With these methods now on firm footing, students then investigate in solution sets of equations joined by “and” or “or” and investigate ways to change an equation such as squaring both sides, which changes the solution set in a controlled (and often useful) way. Next, students learn to use these same skills as they rearrange formulas to define one quantity in terms of another. Finally, students apply all of these new skills and understandings as they work through solving equations and inequalities with two variables including systems of such equations and inequalities.

| Big Idea: | • An equation is a statement of equality between two expressions.  
• An equation with variables is viewed as a question asking for the set of values one can assign to the variables of the equation to make the equation a true statement.  
• Commutative, associate, and distributive properties are identities whose solution sets are the set of all values in the domain of the variables. |
| Essential Questions: | • What limitations are there to the principle “whatever you do to one side of the equation, you must do to the other side?”  
• What must be considered when an equation has a variable in the denominator?  
• How is rearranging formulas the same/different as solving equations that contain a single variable symbol? |
| Vocabulary | Number sentence, algebraic equation, solution set, set notation, identity, inequality, properties of equality, properties of inequality, zero-product property |
| Assessments | Live Binders/Galileo: Topic C Assessment |

### Standard

<table>
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<tr>
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<th>Explanations &amp; Examples</th>
<th>Resources</th>
</tr>
</thead>
</table>

7/2/2014
A.CED.A.3

### A. Create equations that describe numbers or relationships

Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.

#### Example

Create an expression for the right side of each equation such that the solution set for the equation will be all real numbers.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Right Side</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. $4 + 8 = 10 + 5$</td>
<td>$FALSE$</td>
</tr>
<tr>
<td>b. $\frac{1}{2} + \frac{5}{8} = 1.2 - 0.075$</td>
<td>$TRUE$</td>
</tr>
</tbody>
</table>

Which of the following are algebraic equations?

- i. $3.1x - 11.2 = 2.5x + 2.3$
- ii. $10\pi^4 + 3 = 99\pi^2$
- iii. $\pi + \pi = 2\pi$
- iv. $\frac{1}{2} + \pi = \frac{2}{4}$
- v. $79\pi^3 + 70\pi^2 - 56\pi + 87 = \frac{(60\pi + 29,928)}{\pi^2}$

**All of them.**

Create an expression for the right side of each equation such that the solution set for the equation will be all real numbers.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Right Side</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. $2x - 5 = ____$</td>
<td></td>
</tr>
<tr>
<td>b. $x^2 + x = ____$</td>
<td></td>
</tr>
<tr>
<td>c. $4 \cdot x \cdot y \cdot z = ____$</td>
<td></td>
</tr>
<tr>
<td>d. $(x + 2)^2 = ____$</td>
<td></td>
</tr>
</tbody>
</table>

Solve for $a$: $a + a^2 = a(a+1)$. Describe carefully the reasoning that justifies your solution set in words, in set notation, and graphically.

**IN WORDS:** By the distributive property we have $a + a^2 = a(1+a)$. This is
a true numerical statement no matter what value we assign to \( a \). And by the commutative property of addition, we thus have that \( a + a^2 = a(a+1) \) is a true numerical statement no matter what real value we assign to \( a \).

**IN SET NOTATION:** \( R \) (all real numbers)

**IN GRAPHICAL REPRESENTATION:**

![Graphical representation](image)

Solution set in words, set notation and graphically:

| \([-6 \rightarrow -2]\) | \( \{\text{real numbers less than } 4\} \) | \( \{\text{real } | \text{ } z < 4\} \) |

---

**A.CED.A.4**  
**A. Create equations that describe numbers or relationships**

Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law \( V = IR \) to highlight resistance \( R \).

The area \( A \) of a rectangle is 25 in\(^2\). The formula for area is \( A = lw \).

- If the width \( w \) is 10 inches, what is the length \( l \)?
  \[ l = \frac{5}{2} \]

- If the width \( w \) is 15 inches, what is the length \( l \)?
  \[ l = \frac{5}{3} \]

- Rearrange the area formula to solve for \( l \).
  \[ A = lw \]
  \[ \frac{A}{w} = l \text{ or } l = \frac{A}{w} \]
Solve each problem two ways. First, substitute the given values and solve for the given variable. Then, solve for the given variable and substitute the given values.

a. The perimeter formula for a rectangle is \( p = 2(l + w) \) where \( p \) represents the perimeter, \( l \) represents the length, and \( w \) represents the width. Calculate \( l \) when \( p = 70 \) and \( w = 15 \).

   Sample responses:
   Substitute and solve. \( 70 = 2(l + 15) \). \( l = 20 \)
   Solve for the variable first: \( l = \frac{e}{2} - w \)

b. The area formula for a triangle is \( A = \frac{1}{2}bh \), where \( A \) represents the area, \( b \) represents the length of the base, and \( h \) represents the height. Calculate \( b \) when \( A = 100 \) and \( h = 20 \).

   \( b = \frac{2A}{h} \) \( b = 10 \)

---

**Equation Containing More Than One Variable**

Solve \( ax + b = d - cx \) for \( x \).

\[
\begin{align*}
ax + cx + b &= d \\
ax + cx &= d - b \\
x(a + c) &= d - b \\
x &= \frac{d - b}{a + c}
\end{align*}
\]

**Related Equation**

Solve \( 3x + 4 = 6 - 5x \) for \( x \).

\[
\begin{align*}
3x + 5x + 4 &= 6 \\
3x + 5x &= 6 - 4 \\
x(3 + 5) &= 2 \\
8x &= 2 \\
x &= \frac{2}{8} = \frac{1}{4}
\end{align*}
\]
A.REI.A.1

A. Understand solving equations as a process of reasoning and explain the reasoning

Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

A.REI.1

Assuming an equation has a solution, construct a convincing argument that justifies each step in the solution process. Justifications may include the associative, commutative, and division properties, combining like terms, multiplication by 1, etc.

Algebra I, students should focus on and master A.REI.1 for linear equations and be able to extend and apply their reasoning to other types of equations in future courses. Students will solve exponential equations with logarithms in Algebra II.

Strategy taught here:

If we are faced with the task of solving an equation, that is, finding the solution set of the equation:
Use the commutative, associative, distributive properties AND
Use the properties of equality (adding, subtracting, multiplying, dividing by non-zeros) to keep rewriting the equation into one whose solution set you easily recognize. (We observed that the solution set will not change under these operations.)
This usually means rewriting the equation so that all the terms with the variable appear on one side of the equation.

Eureka Math:
Module 1 Lesson 12
Module 1 Lesson 13

Big Ideas:
pp. 3-10
pg 23
Consider the equation $3x^2 + x = (x - 2)(x + 5)x$

a. Use the commutative property to create an equation with the same solution set.
   
   $$x + 3x^2 = (x + 5)(x - 2)x$$

b. Using the result from (a), use the associative property to create an equation with the same solution set.
   
   $$(x + 3x^2) = ((x + 5)(x - 2))x$$

c. Using the result from (b), use the distributive property to create an equation with the same solution set.
   
   $$x + 3x^2 = x^3 + 3x^2 - 10x$$

d. Using the result from (c), add a number to both sides of the equation.
   
   $$x + 3x^2 + 5 = x^3 + 3x^2 - 10x + 5$$

e. Using the result from (d), subtract a number from both sides of the equation.
   
   $$(x + 3x^2 + 5 - 3 = x^3 + 3x^2 - 10x + 5 - 3$$

f. Using the result from (e), multiply both sides of the equation by a number.
   
   $$4(x + 3x^2 + 2) = 4(x^3 + 3x^2 - 10x + 2)$$

g. Using the result from (f), divide both sides of the equation by a number.
   
   $$x + 3x^2 + 2 = x^2 + 3x^2 - 10x + 2$$

h. What do all 7 equations have in common? Justify your answer.
   
   *They will all have the same solution set.*
### A.REI.B.3

**B. Solve equations and inequalities in one variable**

Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x(1 - x) + 2x - 4 = 8x - 24 - x^2 )</td>
<td>Distributive property</td>
</tr>
<tr>
<td>( x - x^2 + 2x - 4 = 8x - 24 - x^2 )</td>
<td>Added ( x^2 ) to both sides of the equation</td>
</tr>
<tr>
<td>( x + 2x - 4 = 8x - 24 )</td>
<td>Collected like terms</td>
</tr>
<tr>
<td>( 3x - 4 = 8x - 24 )</td>
<td>Added 24 to both sides of the equation</td>
</tr>
<tr>
<td>( 3x + 20 = 8x )</td>
<td>Subtracted 8x from both sides of the equation</td>
</tr>
<tr>
<td>( 20 = 5x )</td>
<td></td>
</tr>
</tbody>
</table>

In each of the steps above, we applied a property of real numbers and/or equations to create a new equation.

### Other Resources

- **Eureka Math:**
  - Module 1 Lesson 12
  - Module 1 Lesson 13
  - Module 1 Lesson 14

- **Big Ideas:**
  - pp. 3-10, 11-18, 19-24, 27-34, 61-66, 67-72, 73-78

- **Other:**
  - Learn Zillion
  - The Yo-Yo Problem
  - Reasoning w/ Eq and ineq worksheet
  - Algebra Quiz
  - Texas Instruments:
    - From expressions to Eq
    - Linear inequalities
    - Inequalities on number line
    - Variables on both sides
Just like all our previous work on equations, rewriting an inequality via the commutative, associative, and distributive properties of the real numbers does not change the solution set of that inequality.

\[
\text{Solve } -q \geq -7, \text{ for } q.\\
\begin{align*}
-q &\geq -7 \\
0 &\geq -7 + q \quad \text{Add } q \text{ to both sides} \\
7 &\geq q \quad \text{Add } 7 \text{ to both sides}
\end{align*}
\]

If \( p \geq q \), then \(-p \leq -q\).

\[
\begin{align*}
p &\geq q \\
p - q &\geq q - q \\
p - q &\geq 0 \\
p - p - q &\geq 0 - p \\
-q &\geq -p
\end{align*}
\]

### A.REI.C.5

**C. Solve systems of equations**

Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

Equations of the form \((x - a)(x - b) = 0\) have the same solution set as two equations joined by “or.” \(x - a = 0\) or \(x - b = 0\).

Consider the equation \((x - 4)(x + 3) = 0\).

a. Rewrite the equation as a compound statement.

\[
x - 4 = 0 \text{ or } x + 3 = 0
\]

**Eureka Math:**

Module 1 Lesson 17
Module 1 Lesson 23

**Big Ideas:**

pp. 247-252
Here is a system of two linear equations. Verify that the solution to this system is \((3, 4)\).

Equation A1: \(y = x + 1\)

Equation A2: \(y = -2x + 10\)

Substitute 3 for \(x\) and 4 for \(y\) into both equations.

\[4 = 3 + 1\text{ is a true equation.}\]

\[4 = -2(3) + 10\text{ is a true equation.}\]

Equation D1: \(y = x + 1\)

Equation D2: \(3y = -3x + 21\)

f. What multiple of A2 was added to A1 to create D2?

\(A2\) was multiplied by 2 and then added to A1.

g. What is the solution to the system of two linear equations formed by D1 and D2?

The solution is still \((3, 4)\). I checked by substituting \((3, 4)\) into both equations.

h. Is D2 equivalent to the original A1 or A2? Explain your reasoning.

No. the slope of D2 is \(-1\). Neither of the original equations had that slope.

i. Start with equation A1. Multiply it by a number of your choice and add the result to equation A2. This creates a new equation E2. Record E2 below to check if the solution is \((3, 4)\).

Equation E1: \(y = x + 1\)

Equation E2: \(5y = 2x + 14\)

I multiplied A1 by 4 to get \(4y = 4x + 4\). Adding it to A2 gives \(5y = 2x + 14\). We already know \((3, 4)\) is a solution to \(y = x + 1\). Substituting into E2 gives \(5(4) = 2(3) + 14\), which is a true equation. Therefore \((3, 4)\) is a solution to this new system.
A.REI.C.6

C. Solve systems of equations

Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

Example 1: Why Does the Elimination Method Work?

Solve this system of linear equations algebraically.

**ORIGINAL SYSTEM**

\[ 2x + y = 6 \]
\[ x - 3y = -11 \]

**NEW SYSTEM**

\[ 6x + 3y = 18 \]
\[ x - 3y = -11 \]

**SOLUTION**

Multiply the first equation by 3 and add it to the second. Solve the new system. \((1,4)\)

\[ 7x = 7 \]
\[ 2(1) + y = 6 \text{ so } y = 4 \]

Eureka Math:
Module 1 Lesson 15
Module 1 Lesson 16
Module 1 Lesson 17
Module 1 Lesson 22
Module 1 Lesson 24

Big Ideas:
pp. 235-240 (use problems that replicate the examples that are shown to the left; rather than using the graphing, elimination, substitution approach). Then move to the graphing method.

Texas Instruments:
Balanced Systems
Boats in Motion
For a statement separated by “and” to be true BOTH statements must be true. If it is separated by “or,” at least one statement must be true.

\[ x < 9 \text{ and } x > 7 \]

\[ x + 5 < 7 \text{ or } x = 2 \]

\( (x + 1)(x + 2) = 0 \)
\[ \{ -2, -1 \} \]

\( (x + 4)(x - 6)(x - 10) = 0 \)
\[ \{ -4, 6, 10 \} \]

Students interpret equations like \( \frac{1}{x} = 3 \) as two equations “\( \frac{1}{x} = 3 \)” and “\( x \neq 0 \)” joined by “and.”

**NOTE:** we want to disallow the possibility of ever dividing by zero even \( 0/0 \)!
Consider the equation \( \frac{1}{x} = \frac{3}{x-2} \).

a. Rewrite the equation into a system of equations.
\[
\frac{1}{x} = \frac{3}{x-2} \quad \text{and} \quad x \neq 0 \quad \text{and} \quad x \neq 2
\]

b. Solve the equation for \( x \), excluding the value(s) of \( x \) that lead to a denominator of zero.
\[
x = -1 \quad \text{and} \quad x \neq 0 \quad \text{and} \quad x \neq 2 \quad \text{solution set:} \{-1\}
\]

This is really a compound statement:
\[
\frac{1}{x} = \frac{3}{x-2} \quad \text{and} \quad x \neq 0 \quad \text{and} \quad x - 2 \neq 0
\]

By the properties of equality, we can multiply through by non-zero quantities. Within this compound statement, \( x \) and \( x-2 \) are nonzero, so we may write \( x-2=3x \) and \( x\neq0 \) and \( x-2\neq0 \), which is equivalent to:
\[
-2=2x \quad \text{and} \quad x\neq0 \quad \text{and} \quad x\neq2.
\]
All three declarations in this compound statement are true for \( x=-1 \). This is the solution set.
### D. Represent and solve equations and inequalities graphically

- Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane.

#### Example

Discover as solutions to the equation $4x - y = 10$ as possible. Consider the best way to organize all the solutions you have found.

The sum of two numbers is 25. What are the numbers?

- Create an equation using two variables to represent this relationship.

#### Solution

Solve the following system of equations.

$$\begin{align*}
y &= 2x + 1 \\
x - y &= 7
\end{align*}$$

**Graphically:**

**Algebraically:**

$$x - (2x + 1) = 7$$

$$x = -8$$

$$y = 2(-8) + 1$$

$$y = -15$$

**solution:** $(-8, -15)$

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<table>
<thead>
<tr>
<th>A.REI.D.10</th>
</tr>
</thead>
<tbody>
<tr>
<td>D. Represent and solve equations and inequalities graphically</td>
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<tr>
<td>Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane.</td>
</tr>
</tbody>
</table>

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Eureka Math: Module 1 Lesson 20
Module 1 Lesson 22

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coordinate plane, often forming a curve (which could be a line).

| coordinate plane, often forming a curve (which could be a line). | Let $x = \text{one number, and let } y = \text{another number.}$

\[
\text{Equation: } x + y = 25
\]

- List at least 6 solutions to the equation.
- Create a graph that represents the solution set to the equation.

<table>
<thead>
<tr>
<th>A.REI.D.12</th>
<th>D. Represent and solve equations and inequalities graphically</th>
</tr>
</thead>
<tbody>
<tr>
<td>Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.</td>
<td></td>
</tr>
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</table>

Complete the following sentence: If an ordered pair is a solution to $4x - y \leq 10$, then it will be located on the line or above (or on the left side off) the line $y = 4x - 10$. Explain how you arrived at your conclusion.

I observed that all the points were on one side of the line, and then I tested some points on the other side of the line and found that all the...
points I tested from that side of the line were not solutions to the inequality.

Graph the solution to \( x + y < 20 \)

The line should be dashed, and all points below the line should be shaded.

Graph the solution set to each system of inequalities.

e. \[ \begin{cases} x - y > 5 \\ x > -1 \end{cases} \]

Could you express the solution set of a system of inequalities without using a graph?

- Yes, using set notation, but a graph makes it easier to visualize and conceptualize which points are in the solution set.
How can you check your solution graph?
- *Test a few points to confirm that the points in the shaded region satisfy all the inequalities.*

| MP.1       | Make sense of problems and persevere in solving them. | Students are presented with problems that require them to try special cases and simpler forms of the original problem in order to gain insight into the problem. | Eureka Math:  
Module 1 Lesson 11  
Module 1 Lesson 12  
Module 1 Lesson 14  
Module 1 Lesson 16  
Module 1 Lesson 21  
Module 1 Lesson 24 |
|------------|------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------|
| MP.2       | Reason abstractly and quantitatively.                | Students analyze graphs of non-constant rate measurements and reason from the shape of the graphs to infer what quantities are being displayed and consider possible units to represent those quantities. | Eureka Math:  
Module 1 Lesson 11  
Module 1 Lesson 12  
Module 1 Lesson 14  
Module 1 Lesson 15  
Module 1 Lesson 20 |
| MP.3       | Construct viable arguments and critique the reasoning of others. | Students reason about solving equations using “if-then” moves based on equivalent expressions and properties of equality and inequality. They analyze when an “if-then” move is not reversible. | Eureka Math:  
Module 1 Lesson 11  
Module 1 Lesson 12  
Module 1 Lesson 13  
Module 1 Lesson 16  
Module 1 Lesson 18  
Module 1 Lesson 19  
Module 1 Lesson 24 |
| MP.6       | Attend to precision.                                | Students formalize descriptions of what they learned before (variables, solution sets, numerical expressions, algebraic expressions, etc.) as they build equivalent expressions and solve equations. Students analyze solution sets of equations to determine processes (like squaring both sides of an equation) that might lead to a solution set that differs from that of the original equation. | Eureka Math:  
Module 1 Lesson 15  
Module 1 Lesson 17  
Module 1 Lesson 20  
Module 1 Lesson 24 |
| **MP.7** | **Look for and make use of structure.** | Students reason with and about collections of equivalent expressions to see how all the expressions in the collection are linked together through the properties of operations. They discern patterns in sequences of solving equation problems that reveal structures in the equations themselves: \(2x + 4 = 10\), \(2(x - 3) + 4 = 10\), \(2(3x - 4) + 4 = 10\), etc. | **Eureka Math:** Module 1 Lesson 17 Module 1 Lesson 22 |
| **MP.8** | **Look for and express regularity in repeated reasoning.** | After solving many linear equations in one variable (e.g., \(3x + 5 = 8x - 17\)), students look for general methods for solving a generic linear equation in one variable by replacing the numbers with letters: \(ax + b = cx + d\). They have opportunities to pay close attention to calculations involving the properties of operations, properties of equality, and properties of inequality as they find equivalent expressions and solve equations, noting common ways to solve different types of equations. | **Eureka Math:** Module 1 Lesson 17 |
### Module 1: Relationships Between Quantities and Reasoning with Equations and Their Graphs (40 days)  
**Topic D: Creating Equations to Solve Problems**

In this unit, students are introduced to the modeling cycle (see page 61 of the Common Core Learning Standards) through problems that can be solved using equations and inequalities in one variable, systems of equations, and graphing. Modeling links classroom mathematics and statistics to everyday life, work, and decision making.

**Big Idea:**
- Modeling links classroom mathematics and statistics to everyday life, work, and decision-making.

**Essential Questions:**
- How do I know where to begin when solving a problem?
- How does explaining my process help me to understand a problem’s solution better?
- How do I decide what strategy will work best in a given problem situation?
- What do I do when I get stuck?

**Vocabulary**
- Modeling cycle, (formulate, validate, compute, interpret), recursive sequence, sequence

**Assessments**
- Live Binders/Galileo: Topic D Assessment

<table>
<thead>
<tr>
<th>Standard</th>
<th>Common Core Standards</th>
<th>Explanations &amp; Examples</th>
<th>Resources</th>
</tr>
</thead>
</table>
| N.Q.A.1  | A. Reason qualitatively and units to solve problems | Find three consecutive integers such that their sum is 51. | Eureka Math:  
Module 1 Lesson 25  
Module 1 Lesson 26  
Module 1 Lesson 27  
Module 1 Lesson 28 |

Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
| A.SSE.A.1 | A. Interpret the structure of expressions | Students should understand the vocabulary for the parts that make up the whole expression and be able to identify those parts and interpret their meaning in terms of a context. 

The “such as” listed are not the only parts of an expression students are expected to know; others include, but are not limited to, degree of a polynomial, leading coefficient, constant term, and the standard form of a polynomial (descending exponents). 

The following sequence was generated by an initial value \(a_0\) and recurrence relation \(a_{i+1} = 2a_i + 5\), for \(i \geq 0\).

What is \(a_0\)? What is \(a_3\)?

Using one of the four formulas from Example 1, write an inequality that, if solved for \(a_0\), will lead to finding the smallest starting whole number for the “double and add 5” game that produces a result of 1000 or greater in 3 rounds or less.

- What does \(a_2\) mean in terms of rounds?
  - The result of round two.
- Write what the statement, “produce a result of 1000 or greater in two rounds,” means using a term of the sequence.
  - The result of round two, \(a_2\), must be greater than or equal to 1000. Ask students to write the equation, \(a_2 \geq 1000\), for that statement.
- After replacing \(a_2\) in the inequality, \(a_2 \geq 1000\), with the expression in terms of \(a_0\), what does the numbers \(a_0\) that satisfies the inequality, \(4a_0 + 15 \geq 1000\), mean?
  - The numbers \(a_0\) that satisfies the inequality are the starting numbers for the Double and Add 5 game that produce a result of 1000 or greater in two rounds or less. The “or less” in the previous sentence is important and can be understood by thinking about the question, “Do we need two rounds to reach 1000 starting with number 999? 800? 500?” |

| A.CED.A.1 | A. Create equations that describe numbers | Equations can represent real world and mathematical problems. Include equations and inequalities that arise when comparing the | Eureka Math: Module 1 Lesson 25 |

| | | | Module 1 Lesson 25 Module 1 Lesson 26 Module 1 Lesson 27 Module 1 Lesson 28 |
Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.

values of two different functions, such as one describing linear growth and one describing exponential growth.

Axel and his brother like to play tennis. About three months ago they decided to keep track of how many games they have each won. As of today, Axel has won 18 out of the 30 games against his brother. How many games would Axel have to win in a row in order to have a 75% winning record?

Solving, $18 + n = 0.75(30 + n)$ results in $n = 18$. He would have to win 18 games.

A checking account is set up with an initial balance of $9400, and $800 is removed from the account at the end of each month for rent (no other user transactions occur on the account).

a. Write an inequality whose solutions are the months, $m$, in which the account balance is greater than $3000. Write the solution set to your equation by identifying all of the solutions.

For $m$ a non-negative real number, $m$ satisfies the inequality, $9400 - 800m > 3000$. For real numbers $m$, the solution set is $0 \leq m < 8$.

<table>
<thead>
<tr>
<th>Number</th>
<th>Double and add 5</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_0 = 5$</td>
<td>$5 \cdot 2 + 5 = 15$</td>
<td>$a_0 \cdot 2 + 5 = a_1$</td>
</tr>
<tr>
<td>$a_1 = 15$</td>
<td>$15 \cdot 2 + 5 = 35$</td>
<td>$a_1 \cdot 2 + 5 = a_2$</td>
</tr>
<tr>
<td>$a_2 = 35$</td>
<td>$35 \cdot 2 + 5 = 75$</td>
<td>$a_2 \cdot 2 + 5 = a_3$</td>
</tr>
<tr>
<td>$a_i$</td>
<td>$a_i \cdot 2 + 5 = a_{i+1}$</td>
<td></td>
</tr>
<tr>
<td>$a_{i+1}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Module 1 Lesson 26
Module 1 Lesson 27
Module 1 Lesson 28

Big Ideas:
pp. 6 – 7 ex 3 and 4
pp. 9 -10 #42-44, 47
pg 15 ex 5
pp. 16-17 #29-37
pg. 19
pg 22. Ex 4
pp. 23-24 #27, 28, 33, 34, 39
pg. 53 ex 1
pg. 57 ex 4
pp 58 – 59 #5-14, 25, 26, 46, 47-50, 55-59
pp. 65-66 #26, 29, 32-38
pg 70 ex 3
pp. 71-72 #19-20, 29-32, 35, 38-39
pg. 76 ex 4
pp. 77-78 #31-38
pg 82 ex 1
pg 84 ex 4
pp. 85-86 #3-12, 23-24, 32, 34

Other:
Algebra Balance Scales
Writing and using inequalities – video
Writing and using inequalities
| A.CED.A.2 | A. Create equations that describe numbers or relationships
Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. | Extend the table you created in Exercise 6 by adding a column called, “Effective federal income tax rate.” Compute the effective federal income tax rate to the nearest tenth for each row of the table and create a graph that shows effective federal income tax rate versus income using the table. |
| A.REI.B.3 | B. Solve equations and inequalities in one variable
Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. | The use of tape diagrams and area models are used to solve equations and inequalities given an application problem. The numerical approach is compared to the algebraic approach in investigating real-life situations. |

- Jim tells you he paid a total of $23,078.90 for a car, and you would like to know the price of the car before sales tax so that you can compare the price of that model of car at various dealers. Find price of the car before sales tax if Jim bought the car in:
  - a. Arizona, where the sales tax is 6.6%.
  - Solving \((1+0.066)x=23\,078.90\) results in \(x=21,650\). The car costs $21,650.

- A checking account is set up with an initial balance of $9400, and $800 is removed from the account at the end of each month for rent (no other user transactions occur on the account).
  - a. Write an inequality whose solutions are the months, \(m\), in which the account balance is greater than $3000. Write the solution set to your equation by identifying all of the solutions.
  - For \(m\) a non-negative real number, \(m\) satisfies the inequality, \(9400−800m>3000\). For real numbers \(m\), the solution set is \(0\leq m<8\). |
Find the smallest starting whole number for the *Double and Add 5* game that produces a result of 1,000,000 or greater in four rounds or less.

\[
16 \cdot a_0 + 75 \geq 1,000,000 \\
16a_0 + 75 - 75 \geq 1,000,000 - 75 \\
16a_0 \geq 999,925 \\
\frac{1}{16} (16a_0) \geq \frac{1}{16} (999,925) \\
a_0 \geq \frac{999,925}{16}
\]

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<th><strong>MP.1</strong></th>
<th>Make sense of problems and persevere in solving them.</th>
<th>Students are presented with problems that require them to try special cases and simpler forms of the original problem in order to gain insight into the problem.</th>
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<td>Students analyze graphs of non-constant rate measurements and reason from the shape of the graphs to infer what quantities are being displayed and consider possible units to represent those quantities.</td>
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<td>Eureka Math: Module 1 Lesson 27</td>
</tr>
<tr>
<td><strong>MP.4</strong></td>
<td>Model with mathematics.</td>
<td>Students have numerous opportunities in this module to solve problems arising in everyday life, society, and the workplace - understanding the federal progressive income tax system.</td>
<td>Eureka Math: Module 1 Lesson 26 Module 1 Lesson 27 Module 1 Lesson 28</td>
</tr>
</tbody>
</table>
Module 2: Descriptive Statistics (25 days)

In this module, students reconnect with and deepen their understanding of statistics and probability concepts first introduced in Grades 6, 7, and 8. There is variability in data, and this variability often makes learning from data challenging. Students develop a set of tools for understanding and interpreting variability in data, and begin to make more informed decisions from data. Students work with data distributions of various shapes, centers, and spreads. Measures of center and measures of spread are developed as ways of describing distributions. The choice of appropriate measures of center and spread is tied to distribution shape. Symmetric data distributions are summarized by the mean and mean absolute deviation or standard deviation. The median and the interquartile range summarize data distributions that are skewed. Students calculate and interpret measures of center and spread and compare data distributions using numerical measures and visual representations.

Students build on their experience with bivariate quantitative data from Grade 8; they expand their understanding of linear relationships by connecting the data distribution to a model and informally assessing the selected model using residuals and residual plots. Students explore positive and negative linear relationships and use the correlation coefficient to describe the strength and direction of linear relationships. Students also analyze bivariate categorical data using two-way frequency tables and relative frequency tables. The possible association between two categorical variables is explored by using data summarized in a table to analyze differences in conditional relative frequencies.

This module sets the stage for more extensive work with sampling and inference in later grades.

Topic A: Shapes and Centers of Distributions

In Topic A, students observe and describe data distributions. They reconnect with their earlier study of distributions in Grade 6 by calculating measures of center and describing overall patterns or shapes. Students deepen their understanding of data distributions recognizing that the value of the mean and median are different for skewed distributions and similar for symmetrical distributions. Students select a measure of center based on the distribution shape to appropriately describe a typical value for the data distribution. Topic A moves from the general descriptions used in Grade 6 to more specific descriptions of the shape and the center of a data distribution.

**Big Idea:**

- Statistics is about data.
- Graphs provide a representation of the data distribution and are used to understand the data to answer questions about the distribution.

**Essential Questions:**

- How do various representations of data lead to different interpretations of the data?
- How are center and spread of data sets described and compared?
- How is the choice made between mean and median to describe the typical value related to the shape of the data distribution?
- How do you interpret the mean?
- How are mean and median used to describe distributions of data?

**Vocabulary**

Dot plot, histogram, box plot, mean, skewed data distribution

**Assessments**

Galileo: Algebra 2 Module 1 Foundational Skills Assessment; Live Binders/Galileo: Topic A Assessment

**Standard** | **Common Core Standards** | **Explanations & Examples** | **Resources**
--- | --- | --- | ---
S.ID.A.1 | A. Summarize, represent, and interpret data on a single count or measurement variable | Dot plots: A plot of each data value on a scale or number line. | Eureka Math: Module 2 Lesson 1
Represent data with plots on the real number line (dot plots, histograms, and box plots).

Histograms: A graph of data that groups the data based on intervals and represents the data in each interval by a bar.

Box plots: A graph that provides a picture of the data ordered and divided into four intervals that each contains approximately 25% of the data.

Module 2 Lesson 3

Big Ideas:
pp. 593 – 598
PP. 617 – 622

Illuminations:
Histogram Tool
| S.ID.A.2 | A. Summarize, represent, and interpret data on a single count or measurement variable  
Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. | Eureka Math:  
Module 2 Lesson 2  
Module 2 Lesson 3  

**Big Ideas:**  
Pg. 596  
Pg. 598 #17, 21  
pp. 599-606  

**IXL**  
[Interpret box plots](https://www.ixl.com) |
Calculate the mean number of pets owned by the thirty students from River City High School. Calculate the median number of pets owned by the thirty students.

- The mean number of pets owned is 3.2.
- The median number of pets owned is 2.

S.ID.A.3

A. Summarize, represent, and interpret data on a single count or measurement variable

Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).

What do you think this graph is telling us about the population of the United States?
The percentage of the population is about the same in each interval until the age range of 60 to 65 years old. Then the percentages decline.

What does the box plot tell us about the number of pets owned by the thirty students at River City High School?
- 50% of students own between 1 and 5 pets.

Eureka Math:
Module 2 Lesson 1
Module 2 Lesson 2
Module 2 Lesson 3

Big Ideas:
pp. 585 - 592
pp. 599 – 606

Texas Instruments:
Center and Spread
Describe the distribution of the age of cars.

- The distribution is not symmetric. It is skewed to the left. Most of the data is on the right hand side with a long tail to the left.

What is the mean age of the twenty-five cars? What is the median age? Why are the mean and the median different?

- The mean age is approximately 5.84 years old, and the median age is 7 years old. They are different. The mean is smaller because of the small values in the tail of the data distribution.

Why are the values of the mean and the median that you calculated in question (3) so different? Which of the mean and the median would you use to describe a typical value of coins for these ten students?

What is different about the sophomore data distribution compared to the data distributions for juniors and for seniors?

<table>
<thead>
<tr>
<th>MP.1</th>
<th>Make sense of problems and persevere in solving them.</th>
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<tr>
<td></td>
<td>Students choose an appropriate method of analysis based on problem context. They consider how the data were collected and how data can be summarized to answer statistical questions. Students select a graphical display appropriate to the problem context. They select numerical summaries appropriate to the shape of the data distribution. Students use multiple representations and numerical summaries and then determine the most appropriate representation and summary for a given data distribution. Students are provided three data sets and asked to construct dot plots for each and use them to study the distributions. A symmetrical or a nearly symmetrical distribution emerges from the first data set. Students determine that the mean and median are nearly the same in this distribution. In the second and third data sets, a non-symmetrical distribution is given. Students determine that the mean and the median are not the same. They are asked to explain why the two measures of center are not equal.</td>
</tr>
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<td></td>
<td>Students pose statistical questions and reason about how to collect and interpret data in order to answer these questions. Students form summaries of data using graphs, two-way tables, and other representations that are appropriate for a given context and the statistical question they are trying to answer. Students reason about whether two variables are associated by considering conditional</td>
</tr>
</tbody>
</table>

Interactive: Box Plots

Eureka Math: Module 2 Lesson 2

Eureka Math: Module 2 Lesson 1

Eureka Math: Module 2 Lesson 3
relative frequencies.

- How does the dot plot for the juniors differ from the dot plot for the seniors? What might explain the difference between the dot plots for juniors and seniors?
- How would you describe the typical number of miles walked by a junior?

<table>
<thead>
<tr>
<th>MP.4</th>
<th>Model with mathematics.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Students construct and interpret two-way tables to summarize bivariate categorical data. Students graph bivariate numerical data using a scatterplot and propose a linear, exponential, quadratic, or other model to describe the relationship between two numerical variables. Students use residuals and residual plots to assess if a linear model is an appropriate way to summarize the relationship between two numerical variables. The concept of balance is developed by using a dot plot and a representation of the data as equal weights along a number line. Balance would be a position on the number line in which the sum of the distances on the right and the sum of the distances on the left are equal. Students model with mathematics as they verify this position.</td>
</tr>
</tbody>
</table>

**Eureka Math: Module 2 Lesson 3**
In Topic B, students reconnect with methods for describing variability first seen in Grade 6. Topic B deepens students’ understanding of measures of variability by connecting a measure of the center of a data distribution to an appropriate measure of variability. The mean is used as a measure of center when the distribution is more symmetrical. Students calculate and interpret the mean absolute deviation and the standard deviation to describe variability for data distributions that are approximately symmetric. The median is used as a measure of center for distributions that are more skewed and students interpret the interquartile range as a measure of variability for data distributions that are not symmetric. Students match histograms to box plots for various distributions based on an understanding of center and variability. Students describe data distributions in terms of shape, a measure of center, and a measure of variability from the center.

**Big Idea:**
- The larger the standard deviation, the greater the variability of the data set.
- Data consists of structure plus variability.

**Essential Questions:**
- What measure of center and measure of variability is a better description for skewed distributions?
- Is data distribution represented by the box plot a skewed distribution?
- What information is displayed in a box plot?
- What does standard deviation measure?
- How does the spread of the distribution relate to the value of the standard deviation?

**Vocabulary**
- Deviation from the mean, standard deviation, outliers, interquartile range (IQR), skewed

**Assessments**
- Live Binders/Galileo: Topic B Assessment

<table>
<thead>
<tr>
<th>Standard</th>
<th>Common Core Standards</th>
<th>Explanations &amp; Examples</th>
<th>Resources</th>
</tr>
</thead>
<tbody>
<tr>
<td>S.ID.A.1</td>
<td>A. Summarize, represent, and interpret data on a single count or measurement variable</td>
<td>Ten members of a high school girls’ basketball team were asked how many hours they studied in a typical week. Their responses (in hours) were 20, 13, 10, 6, 13, 10, 13, 11, 11, 10. Using the axis given below, draw a dot plot of these values.</td>
<td>Eureka Math: Module 2 Lesson 4 Module 2 Lesson 5 Module 2 Lesson 6 Module 2 Lesson 7 Big Ideas: Pg 593 exploration #1 Pg 594 ex 1 Pg 601 ex 2</td>
</tr>
</tbody>
</table>
Suppose that every person answers all four questions on the questionnaire. What would the dot plot look like?

Constructing the Box Plot:
Using the above dot plot, construct a box plot over the dot plot by completing the following steps:

i. Locate the middle 40 observations, and draw a box around these values.
ii. Calculate the median, and then draw a line in the box at the location of the median.
iii. Draw a line that extends from the upper end of the box to the largest observation in the data set.
iv. Draw a line that extends from the lower edge of the box to the minimum value in the data set.
<table>
<thead>
<tr>
<th>S.ID.A.2</th>
<th>A. Summarize, represent, and interpret data on a single count or measurement variable</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.</td>
</tr>
</tbody>
</table>

Remember that the size of the standard deviation is related to the sizes of the deviations from the mean. Without doing any calculations, indicate which of the three races has the smallest standard deviation of times. Justify your answer.

Which race had the largest standard deviation of times? (Again, don’t do any calculations!) Justify your answer.

Roughly what would be the standard deviation in Race 1? (Remember that the standard deviation is a typical deviation from the mean. So, here you are looking for a typical deviation from the mean, in seconds, for Race 1.)

Use your calculator to find the mean and the standard deviation for each of the three races. Write your answers in the table below to the nearest thousandth.

Comparing histograms:
How do the shapes of the two histograms differ?

Comparing Box Plots:
What information is displayed in a box plot?
Can we find this same information in the histograms?
Why is the mean _____ greater than the median ____? |
### A. Summarize, represent, and interpret data on a single count or measurement variable

Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).

Jenna has bought a new hybrid car. Each week for a period of seven weeks, she has noted the fuel efficiency (in miles per gallon) of her car. The results are shown below.

<table>
<thead>
<tr>
<th>Miles per Gallon</th>
</tr>
</thead>
<tbody>
<tr>
<td>45</td>
</tr>
<tr>
<td>44</td>
</tr>
<tr>
<td>43</td>
</tr>
<tr>
<td>44</td>
</tr>
<tr>
<td>45</td>
</tr>
<tr>
<td>44</td>
</tr>
<tr>
<td>43</td>
</tr>
</tbody>
</table>

Calculate the standard deviation of these results to the nearest hundredth. Be sure to show your work.

- **The mean is 44.**
- **The deviations from the mean are** 1, −1, 0, 1, −1.
- **The squared deviations from the mean are** 1, 1, 0, 1.
- **The sum of the squared deviations is** 4. $n = 7; s^2 = 0.667$
- **The standard deviation is** $s = \sqrt{0.667} \approx 0.82$ miles per gallon.

What is the meaning of the standard deviation you found?

**The standard deviation, 0.82 miles per gallon, is a typical deviation of a weekly fuel efficiency value from the mean weekly fuel efficiency.**

Why do we take the square root?

Why did we divide by $n − 1$ instead of $n$?

What does standard deviation measure?

How can we summarize what we are attempting to compute?

---

**Interpreting the Box Plot:**

- **Minimum age:** 5
- **Lower quartile or Q1:** 40
- **Median Age:** 60
- **Upper quartile or Q3:** 70
- **Maximum age:** 75

What percent of the data does the box part of the box plot capture?

**The box captures 50% of the viewers.**

What percent of the data falls between the minimum value and Q1?

**25% of the viewers fall between the minimum value and Q1.**

What percent of the data falls between Q3 and the maximum value?

**25% of the viewers fall between Q3 and the maximum value.**

What is the interquartile range (IQR) for this distribution? What
<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
</table>
| **MP.1** | **Make sense of problems and persevere in solving them.** | Students choose an appropriate method of analysis based on problem context. They consider how the data were collected and how data can be summarized to answer statistical questions. Students select a graphical display appropriate to the problem context. They select numerical summaries appropriate to the shape of the data distribution. Students use multiple representations and numerical summaries and then determine the most appropriate representation and summary for a given data distribution. | **Eureka Math:**  
Module 2 Lesson 4  
Module 2 Lesson 6 |
| **MP.2** | **Reason abstractly and quantitatively.** | Students pose statistical questions and reason about how to collect and interpret data in order to answer these questions. Students form summaries of data using graphs, two-way tables, and other representations that are appropriate for a given context and the statistical question they are trying to answer. Students reason about whether two variables are associated by considering conditional relative frequencies. | **Eureka Math:**  
Module 2 Lesson 6 |
| **MP.3** | **Construct viable arguments and critique the reasoning of others.** | Students examine the shape, center, and variability of a data distribution and use characteristics of the data distribution to communicate the answer to a statistical question in the form of a poster presentation. Students also have an opportunity to critique poster presentations made by other students. | **Eureka Math:**  
Module 2 Lesson 4  
Module 2 Lesson 5  
Module 2 Lesson 7  
Module 2 Lesson 8 |
In Topic C, students reconnect with previous work in Grade 8 involving categorical data. Students use a two-way frequency table to organize data on two categorical variables. Students calculate the conditional relative frequencies from the frequency table. They explore a possible association between two categorical variables using differences in conditional relative frequencies. Students also come to understand the distinction between association between two categorical variables and a causal relationship between two variables. This provides a foundation for work on sampling and inference in later grades.

**Big Idea:**
- Data in a two-way frequency table can be summarized using relative frequencies in the context of the data.
- Association does not imply causation.

**Essential Questions:**
- What is the difference between categorical data and numerical data?
- How is relative frequency calculated?

**Vocabulary**
- Two-way frequency table, marginal frequency, categorical data, numerical data, joint frequency, relative frequency table, conditional relative frequency

**Assessments**
- Live Binders/Galileo: Topic C Assessment

**Standard**

<table>
<thead>
<tr>
<th>Common Core Standards</th>
<th>Explanations &amp; Examples</th>
<th>Resources</th>
</tr>
</thead>
<tbody>
<tr>
<td>S.ID.B.5</td>
<td>B. Summarize, represent, and interpret data on a single count or measurement variable</td>
<td>The data used in this lesson were obtained from the Census at School project. Census at School is further explained at the American Statistical Association website* and can be a source for data that might interest teenagers. *<a href="http://www.amstat.org/censusatschool">www.amstat.org/censusatschool</a></td>
</tr>
</tbody>
</table>

Another random sample of 100 surveys was selected. Jill had a copy of the frequency table that summarized these 100 surveys. Unfortunately, she spilled part of her lunch on the copy. The following summaries were still readable:

<table>
<thead>
<tr>
<th></th>
<th>To Fly</th>
<th>Freeze Time</th>
<th>Invisibility</th>
<th>Super Strength</th>
<th>Telepathy</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Females</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>55</td>
</tr>
<tr>
<td></td>
<td>12</td>
<td>15</td>
<td>(c)^</td>
<td>5</td>
<td>(c)^</td>
<td></td>
</tr>
<tr>
<td>Males</td>
<td>12</td>
<td>16</td>
<td>10</td>
<td>(g)^</td>
<td>3</td>
<td>45</td>
</tr>
<tr>
<td>Total</td>
<td>24</td>
<td>31</td>
<td>25</td>
<td>9</td>
<td>(g)^</td>
<td>100</td>
</tr>
</tbody>
</table>
Help Jill recreate the table by determining the frequencies for cells (c), (e), (j), and (q). (c) has 15 students, (e) has 8 students, (j) has 4 students, and (q) has 11 students.

Of the cells (c), (e), (j), and (q), which cells represent joint frequencies? *The cells (c), (e), and (j) are joint frequencies.*

Of the cells (c), (e), (j), and (q), which cells represent marginal frequencies? *Cell (q) is a marginal frequency.*

**S.ID.C.9**

**C. Interpret linear models**

Distinguish between correlation and causation.

The focus of this lesson is conditional relative frequencies and how they indicate a possible association. Definitions of conditional relative frequencies and association are provided in this lesson. Differences in conditional relative frequencies are used as evidence of possible association. Instructors should challenge students to think critically about the meaning of an association between two categorical variables and to be careful not to draw unwarranted conclusions about possible cause-and-effect relationships between two categorical variables. The last example and exercise discuss the issue of cause-and-effect.

Discuss other examples of questions that investigate association:
- If dogs are classified as large, medium, or small based on weight, are small dogs more likely to pass an obedience course?
- If users of a social network are classified as active, average, or inactive, is a person classified as an *active* user more likely to be a good writer than those classified in the other categories?

Then discuss the evidence that supports association:
- There is strong evidence of association when there is a *noticeable* difference in conditional relative frequencies.
- What is a *noticeable* difference in conditional relative frequencies? This is subjective; students should use their best judgment at this time. Evaluating the differences more formally is discussed in the Grade 11 and 12 modules.

Students were given the opportunity to prepare for a college placement test in mathematics by taking a review course. Not all students took advantage of this opportunity. The following results were obtained from a random sample of students who took the placement test:
This example will introduce the important idea that you should not infer a cause-and-effect relationship from an association between two categorical variables. Read through the example with students. Pose the following questions to the class. Let students discuss their ideas.

- Do you think there is an association between taking the review course and a student’s placement in a math class?
- If you knew that a student took a review course, would it make a difference in what you predicted for which math course they were placed in?
- Do you think taking a course caused a student to place higher in a math placement?

<table>
<thead>
<tr>
<th>Took Review Course</th>
<th>Placed in Math 200</th>
<th>Placed in Math 100</th>
<th>Placed in Math 50</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>13</td>
<td>7</td>
<td></td>
<td>60</td>
</tr>
<tr>
<td>10</td>
<td>15</td>
<td>15</td>
<td></td>
<td>40</td>
</tr>
<tr>
<td>Total</td>
<td>50</td>
<td>28</td>
<td>22</td>
<td>100</td>
</tr>
</tbody>
</table>

**MP.1 Make sense of problems and persevere in solving them.**

Students choose an appropriate method of analysis based on problem context. They consider how the data were collected and how data can be summarized to answer statistical questions. Students select a graphical display appropriate to the problem context. They select numerical summaries appropriate to the shape of the data distribution. Students use multiple representations and numerical summaries and then determine the most appropriate representation and summary for a given data distribution.
**HS Algebra I Semester 1**

**Module 2: Descriptive Statistics (25 days)**

**Topic D: Numerical Data on Two Variables**

In Topic D, students analyze relationships between two quantitative variables using scatterplots and by summarizing linear relationships using the least squares regression line. Models are proposed based on an understanding of the equations representing the models and the observed pattern in the scatter plot. Students calculate and analyze residuals based on an interpretation of residuals as prediction errors.

**Big Idea:**
- Synthesizing information from multiple sets of data results in evidence-based interpretation.
- A relationship between two numerical variables can be described as a linear or nonlinear relationship.
- Models can be used to answer questions about how two variables are related.

**Essential Questions:**
- Why are multiple sets of data used?
- How can a graph be used to make a prediction? How can a model be used to make a prediction?
- Why does correlation no imply causation?
- How do the patterns in a residual plot show us whether a linear model is a good fit for the data?
- How are residuals used to evaluate the accuracy of predictions based on the least-squares line?

**Vocabulary**
- Residuals,
- the least-squares line (best-fit line), residual plot, correlation coefficient, causation

**Assessments**
- Live Binders/Galileo: Topic D Assessment

**Standard**

<table>
<thead>
<tr>
<th><strong>Common Core Standards</strong></th>
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<th><strong>Resources</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>S.ID.B.6</strong></td>
<td>B. Summarize, represent, and interpret data on a single count or measurement variable</td>
<td>Tasks have a real-world context.</td>
</tr>
<tr>
<td></td>
<td>Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.</td>
<td><strong>Algebra I</strong> students take a more sophisticated look at using a linear function to model the relationship between two numerical variables. In addition to fitting a line to data, students assess how well the model fits by analyzing residuals. <strong>S.ID.6a</strong> – tasks have a real-world context. In Algebra I, exponential functions are limited to those with domains in the integers.</td>
</tr>
<tr>
<td></td>
<td>a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or chooses a function suggested by the context. Emphasize linear, quadratic, and exponential models.</td>
<td>One way to think about how useful a line is for describing a relationship between two variables is to use the line to predict the y values for the points in the scatter plot. These predicted values could then be compared to the actual y values. For example, the first data point in the table represents a man with a shoe length of</td>
</tr>
<tr>
<td></td>
<td>b. Informally assess the fit of a function by plotting and analyzing residuals.</td>
<td></td>
</tr>
</tbody>
</table>

**Eureka Math:**
- Module 2 Lesson 12
- Module 2 Lesson 13
- Module 2 Lesson 14
- Module 2 Lesson 15
- Module 2 Lesson 16
- Module 2 Lesson 17
- Module 2 Lesson 20

**Big Ideas:**
- a. pp. 195, 198, 200, 201-
c. Fit a linear function for a scatter plot that suggests a linear association.

12.6 inches and height of 74 inches. If you use the line \( y = 25.3 + 3.66x \) to predict this man's height, you would get:
\[
y = 25.3 + 3.66x \\
= 25.3 + 3.66(12.6) \\
= 71.42 \text{ inches}
\]

Because his actual height was 74 inches, you can calculate the prediction error by subtracting the predicted value from the actual value. This prediction error is called a residual. For the first data point, the residual is calculated as follows:

\[
\text{Residual} = \text{actual } y \text{ value} - \text{predicted } y \text{ value} \\
= 74 - 71.42 \\
= 2.58 \text{ inches}
\]

Finding the Regression Line (TI-84 Plus)

Step 1: From your home screen, press STAT.
Step 2: From the STAT menu, select the EDIT option. (EDIT enter)
Step 3: Enter the x-values of the data set in L1.
Step 4: Enter the y-values of the data set in L2.
Step 5: Select STAT. Move cursor to the menu item CALC and then move the cursor to option 4: LinReg(\(a+bx\)) or option 8: LinReg(\(ax+b\)). Press enter. (Discuss with students that both options 4 and 8 are representations of a linear equation. It is anticipated that most students are familiar with option 4, or the slope-\(y\)-intercept form. Option 8 is essentially the same representation using different letters to represent slope and \(y\)-intercept. Option 8 is the preferred option in statistical studies.)
Step 6: With option 4 or option 8 on the screen, enter L1, L2, and Y1 as described in the following notes.

LinReg(\(a+bx\)) L1, L2, Y1
Select enter to see results. The least-squares regression will be stored in Y1. Work with students in graphing the scatter plot and Y1.

Note: L1 represents the \(x\)-values of the regression function, L2 the \(y\)-values, and Y1 represents the least squares regression function.

To obtain Y1, go to VARS, move cursor to Y-VARS, and then Functions (enter). You are now at the screen highlighting the \(y\)-variables. Move cursor to Y1 and hit enter.

Y1 is the linear regression line, and will be stored in Y1.

Construction of residual plot:
1. From the home screen, press 2nd, STATPLOT.
2. Select Plot2 and press ENTER.
3. Select “On”, under “Type” choose the first (scatter plot) icon, for Xlist enter L1, for Ylist enter RESID, and under “Mark” choose the first (square) symbol. (“RESID” is accessed by pressing 2nd, LIST, selecting NAMES, scrolling down to RESID, and pressing ENTER.)
4. Press 2nd, QUIT to return to the home screen.
5. Press Y=.
6. First, deselect the equation of the least-squares line in Y1 by going to the “=” sign for Y1 and pressing ENTER. Then deselect Plot1 and make sure that Plot2 is selected.
7. Press Zoom, select ZoomStat (option 9), press ENTER.
8. The residual plot is displayed.

What does the residual plot indicate about using a linear model?
C. Interpret linear models

Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.

Once you have found the equation of the least-squares line, the values of the slope and y-intercept of the line often reveals something interesting about the relationship you are modeling.

The slope of the least-squares line is the change in the predicted value of the y variable associated with an increase of one in the value of the x-variable.

Give an interpretation of the slope of the least-squares line \( y = 25.3 + 3.66x \) for predicting height from shoe size for adult men.

The slope is 3.66; for every one-inch increase in a man’s shoe length, you would estimate an increase of 3.66 inches in height.

The y-intercept of a line is the predicted value of y when x equals zero. When using a line as a model for the relationship between two numerical variables, it often does not make sense to interpret the y-intercept because an x-value of zero may not make any sense.

Explain why it does not make sense to interpret the y-intercept of 25.3 as the predicted height for an adult male whose shoe length is zero.

The y-intercept is \((0, 25.3)\). Since x represents shoe length, it is impossible for the shoe length to be 0 inches when a man is 25.3 inches tall.

S.ID.C.7

S.ID.C.8

C. Interpret linear models

Compute (using technology) and interpret the correlation coefficient of a linear fit.

S.ID.8 Use a calculator or computer to find the correlation coefficient for a linear association. Interpret the meaning of the value in the context of the data.

Build on students’ work with linear relationships in eighth grade and introduce the correlation coefficient. The focus here is on the computation and interpretation of the correlation coefficient as a measure of how well the data fit the relationship.

The correlation coefficient is a number between \(-1\) and \(+1\) (including \(-1\) and \(+1\)) that measures the strength and direction of a linear relationship. The correlation coefficient is denoted by the letter \(r\).

Property 1: The sign of \(r\) (positive or negative) corresponds to the direction of the linear relationship

Property 2: A value of \(r = +1\) indicates a perfect positive linear relationship, with all points in the scatter plot falling exactly on a straight line.
Property 3: A value of \( r = -1 \) indicates a perfect negative linear relationship, with all points in the scatter plot falling exactly on a straight line.

Property 4: The closer the value of \( r \) is to +1 or −1, the stronger the linear relationship.

Steps for calculating the correlation coefficient using a TI-84 Plus

Step 1: Determine which variable represents \( x \), and which variable represents \( y \) based on \( x \) and \( y \) variable designations.

Step 2: From the home screen, select STAT.

Step 3: Click ENTER from the Edit option of the menu.

Step 4: Enter the values of \( x \) in L1 and the values of \( y \) in L2.

Step 5: When complete, enter 2nd QUIT.

Step 6: Select STAT.

Step 7: With the arrows, move the top cursor over to the option CALC and move the down cursor to 8: LinReg(\( a+bx \)), and then click ENTER.

Step 8: With LinReg(\( a+bx \)) on the screen, enter L1, L2 and then ENTER.

Step 9: The value of \( r \), the correlation coefficient, should appear on the screen.

Given a set of data:

- Based on the scatter plot, do you think that the value of the correlation coefficient between ______ and _____ will be positive or negative? Explain why you made this choice.
- Based on the scatter plot, estimate the value of the correlation coefficient between _____ and ______.
- Calculate the value of the correlation coefficient between _____ and ______. Round to the nearest hundredth. Interpret this value.

Is there a connection between the slope of the least-squares line and the value of the correlation coefficient or \( r \)? If yes, what is the connection?

There is a connection regarding the sign of \( r \) (the correlation coefficient) and the sign of the slope if there is a linear relationship. If the least-squares line is increasing, the slope is positive and the value of the correlation coefficient, or \( r \), is positive. If the least-squares line is decreasing, then the slope is negative and the value of the correlation coefficient is negative.

Why is it important to know if a relationship is strong or weak?
If a relationship is strong, then the data are close to the line, and the equation of the line can be used to predict values. If the relationship is weak, then the equation cannot be used as easily to predict values.

S.ID.C.9  
C. Interpret linear models  
Distinguish between correlation and causation.

The important distinction between a statistical relationship and a cause-and-effect relationship arises in S.ID.9. It is sometimes tempting to conclude that if there is a strong linear relationship between two variables that one variable is causing the value of the other variable to increase or decrease. But you should avoid making this mistake. When there is a strong linear relationship, it means that the two variables tend to vary together in a predictable way, which might be due to something other than a cause-and-effect relationship. For example, the value of the correlation coefficient between sodium content and number of calories for the fast food items in an example was $r=0.79$, indicating a strong positive relationship. This means that the items with higher sodium content tend to have a higher number of calories. But the high number of calories is not caused by the high sodium content. In fact sodium does not have any calories. What may be happening is that food items with high sodium content also may be the items that are high in sugar and/or fat, and this is the reason for the higher number of calories in these items. Similarly, there is a strong positive correlation between shoe size and reading ability in children. But it would be silly to think that having big feet causes children to read better. It just means that the two variables vary together in a predictable way. Can you think of a reason that might explain why children with larger feet also tend to score higher on reading tests?

MP.1  
Make sense of problems and persevere in solving them.

Students choose an appropriate method of analysis based on problem context. They consider how the data were collected and how data can be summarized to answer statistical questions. Students select a graphical display appropriate to the problem context. They select numerical summaries appropriate to the shape of the data distribution. Students use multiple representations and numerical summaries and then determine the most appropriate representation and summary for a given data distribution.

MP.2  
Reason abstractly and quantitatively.

Students pose statistical questions and reason about how to collect and interpret data in order to answer these questions. Students form summaries of data using graphs, two-way tables, and other representations that are appropriate for a given context and the statistical question they are trying to answer. Students reason about whether two variables are associated by considering conditional relative frequencies.
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<tr>
<th>MP.3</th>
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<th>Students examine the shape, center, and variability of a data distribution and use characteristics of the data distribution to communicate the answer to a statistical question in the form of a poster presentation. Students also have an opportunity to critique poster presentations made by other students.</th>
<th>Eureka Math: Module 2 Lesson 18 Module 2 Lesson 20</th>
</tr>
</thead>
<tbody>
<tr>
<td>MP.4</td>
<td>Model with mathematics.</td>
<td>Students construct and interpret two-way tables to summarize bivariate categorical data. Students graph bivariate numerical data using a scatterplot and propose a linear, exponential, quadratic, or other model to describe the relationship between two numerical variables. Students use residuals and residual plots to assess if a linear model is an appropriate way to summarize the relationship between two numerical variables.</td>
<td>Eureka Math: Module 2 Lesson 12 Module 2 Lesson 13 Module 2 Lesson 14 Module 2 Lesson 15 Module 2 Lesson 16</td>
</tr>
<tr>
<td>MP.5</td>
<td>Use appropriate tools strategically.</td>
<td>Students visualize data distributions and relationships between numerical variables using graphing software. They select and analyze models that are fit using appropriate technology to determine whether or not the model is appropriate. Students use visual representations of data distributions from technology to answer statistical questions.</td>
<td>Eureka Math: Module 2 Lesson 17</td>
</tr>
<tr>
<td>MP.6</td>
<td>Attend to precision.</td>
<td>Students interpret and communicate conclusions in context based on graphical and numerical data summaries. Students use statistical terminology appropriately.</td>
<td>Eureka Math: Module 2 Lesson 16</td>
</tr>
</tbody>
</table>