### 8th Grade Algebra I Semester 2 (Quarter 3)

#### Unit 3: Polynomial and Quadratic Expressions, Equations and Functions (30 days)

**Topic A: Quadratic Expressions, Equations, Functions, and Their Connection to Rectangles (10 days)**

By the end of middle school, students are familiar with linear equations in one variable (6.EE.B.5, 6.EE.B.6, 6.EE.B.7) and have applied graphical and algebraic methods to analyze and manipulate equations in two variables (7.EE.A.2). They used expressions and equations to solve real-life problems (7.EE.B.4). They have experience with square and cube roots, irrational numbers (8.NS.A.1), and expressions with integer exponents (8.EE.A.1).

In Grade 9, students have been analyzing the process of solving equations and developing fluency in writing, interpreting, and translating between various forms of linear equations (Unit 1) and linear and exponential functions (Unit 2). These experiences set the stage for Module 4. Here students continue to interpret expressions, create equations, rewrite equations and functions in different but equivalent forms, and graph and interpret functions, but this time using polynomial functions, and more specifically quadratic functions, as well as square root and cube root functions.

Topic A introduces polynomial expressions. In Unit 1, students learned the definition of a polynomial and how to add, subtract, and multiply polynomials. Here their work with multiplication is extended and then, connected to factoring of polynomial expressions and solving basic polynomial equations (A-APR.A.1, A-REI.D.11). They analyze, interpret, and use the structure of polynomial expressions to multiply and factor polynomial expressions (A-SSE.A.2). They understand factoring as the reverse process of multiplication. In this topic, students develop the factoring skills needed to solve quadratic equations and simple polynomial equations by using the zero-product property (A-SSE.B.3a). Students transform quadratic expressions from standard or extended form, \(ax^2+bx+c\), to factored form and then solve equations involving those expressions. They identify the solutions of the equation as the zeros of the related function. Students apply symmetry to create and interpret graphs of quadratic functions (F-IF.B.4, F-IF.C.7a). They use average rate of change on an interval to determine where the function is increasing/decreasing (F-IF.B.6). Using area models, students explore strategies for factoring more complicated quadratic expressions, including the product-sum method and rectangular arrays. They create one- and two-variable equations from tables, graphs, and contexts and use them to solve contextual problems represented by the quadratic function (A-CED.A.1, A-CED.A.2) and relate the domain and range for the function, to its graph, and the context (F-IF.B.5).

**Big Idea:**
- Factoring is the reverse process of multiplication.
- Multiplying binomials is an application of the distributive property; each term in the first binomial is distributed over the terms of the second binomial.
- The area model can be modified into a tabular form to model the multiplication of binomials (or other polynomials) that may involve negative terms.
- Quadratic functions create a symmetrical curve with its highest or lowest point corresponding to its vertex and an axis of symmetry passing through it when graphed.

**Essential Questions:**
- Why is the final result when you multiply two binomials sometimes only three terms?
- How can we know whether a graph of a quadratic function will open up or down?
- How are finding the slope of a line and finding the average rate of change on an interval of a quadratic function similar? Different?
- Why is the leading coefficient always negative for functions representing falling objects?

**Vocabulary**
- Binomial, expanding, polynomial expression, quadratic expression, product-sum method, splitting the linear term, tabular model, axis of symmetry, vertex, end behavior of a graph, rate of change

**Assessment**
- Galileo: Module 4 Foundational Skills Assessment; Topic A Assessment

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6/12/2016
A. SSE.A.1

A. Interpret the structure of expressions

a. Interpret parts of an expression, such as terms, factors, and coefficients.

b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret $P(1+r)^n$ as the product of $P$ and a factor not depending on $P$.

This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.

Explanation:
This standard is taught in Algebra I and Algebra II. In Algebra I the focus is on linear expressions, exponential expressions with integer exponents and quadratic expressions. Throughout Algebra I, students should:

- **Explain** the difference between an expression and an equation.
- **Use** appropriate vocabulary for the parts that make up the whole expression.
- **Identify** the different parts of the expression and explain their meaning within the context of the problem.
- **Decompose** expressions and make sense of the multiple factors and terms by explaining the meaning of the individual parts.

Note: Students should understand the vocabulary for the parts that make up the whole expression, be able to identify those parts, and interpret their meaning in terms of a context.

a. Interpret parts of an expression, such as: terms, factors, and coefficients

- Students recognize that the linear expression $mx + b$ has two terms, $m$ is a coefficient, and $b$ is a constant.
- Students extend beyond simplifying an expression and address interpretation of the components in an algebraic expression.
- Development and proper use of mathematical language is an important building block for future content. Using real-world context examples, the nature of algebraic expressions can be explored.
- The “such as” listed are not the only parts of an expression students are expected to know; others include, but are not limited to, degree of a polynomial, leading coefficient, constant term, and the standard form of a polynomial (descending exponents).

Examples:
A student recognizes that in the expression \(2x + 1\), “2” is the coefficient, “2” and “x” are factors, and “1” is a constant, as well as “2x” and “1” being terms of the binomial expression.

A student recognizes that in the expression \(4(3)^x\), 4 is the coefficient, 3 is the factor, and \(x\) is the exponent.

The height (in feet) of a balloon filled with helium can be expressed by \(5 + 6.3s\) where \(s\) is the number of seconds since the balloon was released. Identify and interpret the terms and coefficients of the expression.

A company uses two different sized trucks to deliver sand. The first truck can transport \(x\) cubic yards, and the second \(y\) cubic yards. The first truck makes \(S\) trips to a job site, while the second makes \(T\) trips. What do the following expressions represent in practical terms?

\[
a. \ S + T \\
b. \ x + y \\
c. \ xS + yT
\]

b. Interpret complicated expressions by viewing one or more of their parts as a single entity.

- Students view \(mx\) in the expression \(mx + b\) as a single quantity.

Examples:

- The expression \(20(4x) + 500\) represents the cost in dollars of the materials and labor needed to build a square fence with side length \(x\) feet around a playground. Interpret the constants and coefficients of the expression in context.

- A rectangle has a length that is 2 units longer than the width. If the width is increased by 4 units and the length increased by 3 units, write two equivalent expression for the area of the rectangle.

  - The area of the rectangle is \((x+5)(x+4) = x^2 + 9x + 20\). Students should recognize \((x+5)\) as the length of the modified rectangle and \((x+4)\) as the width. Students can also interpret \(x^2 + 9x + 20\) as the sum of the three areas (a square with side length \(x\), a rectangle with side lengths 9 and \(x\), and another rectangle with area \(x\)).
20 that have the same total area as the modified rectangle.

- Consider the expression $4000p - 250p^2$ that represents income from a concert where $p$ is the price per ticket. The equivalent factored form, $p(4000 - 250p)$, shows that the income can be interpreted as the price times the number of people in attendance based on the price charged. Students recognize $(4000 - 250p)$ as a single quantity for the number of people in attendance.

- The expression $10,000(1.055)^n$ is the amount of money in an investment account with interest compounded annually for $n$ years. Determine the initial investment and the annual interest rate.

  **Note:** the factor of 1.055 can be rewritten as $(1 + 0.055)$, revealing the growth rate of 5.5% per year.

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**A.SSE.A.2**

**A. Interpret the structure of expressions**

Use the structure of an expression to identify ways to rewrite it. For example, see $x^2 - y^2$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.

**Explanation:**

This standard is taught in Algebra I and Algebra II. In Algebra I tasks are limited to numerical and polynomial expressions in one variable, with a focus on quadratics. Examples: Recognize that $53^2 - 47^2$ is the difference of squares and see an opportunity to rewrite it in the easier-to-evaluate form $(53 - 47)(53 + 47)$. See an opportunity to rewrite $a^2 + 9a + 14$ as $(a + 7)(a + 2)$. Can include the sum or difference of cubes (in one variable), and factoring by grouping.

Use factoring techniques such as common factors, grouping, the difference of two squares, or a combination of methods to factor quadratics completely. Students should extract the greatest common factor (whether a constant, a variable or a combination of each). If the remaining expressions is a factorable quadratic, students should factor the expression further.

If the leading coefficient for a quadratic expression is not 1, the first step in factoring should be to see if all the terms in the expanded form have a common factor. Then after factoring out the greatest common factor, it may be possible to factor again.

**Examples:**
Factor $2x^3 - 50x$ completely:

The GCF of the expression is $2x$, $2x(x^2 - 25)$

Now factor the difference of squares: $2x(x - 5)(x + 5)$

- Factor completely: $3t^3 + 18t^2 - 48t$
- Factor completely: $4n - n^3$

$16a^2b^4 + 20ab^2 - 6$

$(4ab^2)^2 + 5(4ab^2) - 6$

$(4ab^2 + 6)(4ab^2 - 1)$

Factored form: $2(2ab^2 + 3)(4ab^2 - 1)$

A.SSE.B.3a

B. Write expressions in equivalent forms to solve problems

Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

a. Factor a quadratic expression to reveal the zeros of the function it defines.

This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.

Explanation:

Students write expressions in equivalent forms by factoring to find the zeros of a quadratic function and explain the meaning of the zeros.

Examples:

- Given a quadratic function explain the meaning of the zeros of the function. That is if $f(x) = (x - c)(x - a)$ then $f(a) = 0$ and $f(c) = 0$.
- Given a quadratic expression, explain the meaning of the zeros graphically. That is for an expression $(x - a)(x - c)$, a and c correspond to the x-intercepts (if a and c are real).
- The expression $-5x^2 + 20x - 15$ represents the height of a ball in meters as it is thrown from one person to another where $x$ is the number of seconds.
  - Rewrite the expression to reveal the linear factors.
  - Identify the zeros of the expression and interpret what they mean in regards to the context.
  - How long is the ball in the air?

Eureka Math:
Module 4 Lesson 7
A.APR.A.1

A. Perform arithmetic operations on polynomials
Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

Explanation:
The primary strategy for this cluster is to make connections between arithmetic of integers and arithmetic of polynomials. In order to understand this standard, students need to work toward both understanding and fluency with polynomial arithmetic. Furthermore, to talk about their work, students will need to use correct vocabulary, such as integer, monomial, polynomial, factor, and term.

Examples: (refer to examples from Unit 1 in addition to the examples below)
- Multiply (x+2) and (x+5)

\[
\begin{array}{c|c|c}
  & x & 5 \\
\hline
 x & x^2 & 5x \\
 x + 2 & 2x & 10 \\
\hline
\end{array}
\]

\[x^2 + 7x + 10\]

Eureka Math: Module 4 Lesson 1, 2

A garden measuring 12 m by 16 m is to have a pedestrian pathway that is \(w\) meters wide installed all the way around it, increasing the total area to 285 sq. m. What is the width, \(w\), of the pathway?

\[(12 + w)(16 + w) = 285\]
\[w^2 + 28w - 93 = 0\]
\[(w + 31)(w - 3) = 0\]
\[w = 3 \text{ or } -31\]

However, only the positive value makes sense in this context, so \(w = 3\).
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<th>B. Understand the relationship between zeros and factors of polynomials.</th>
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<td>Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.</td>
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</table>

**Explanation:**
This standard is taught in Algebra I and Algebra II. In Algebra I, tasks are limited to quadratic and cubic polynomials, in which linear and quadratic factors are available. For example, find the zeros of \((x – 2)\left(x^2 – 9\right)\).

**Examples:**

A science class designed a ball launcher and tested it by shooting a tennis ball straight up from the top of a 15-story building. They determined that the motion of the ball could be described by the function:

\[ h(t) = -16t^2 + 144t + 160, \]

where \(t\) represents the time the ball is in the air in seconds and \(h(t)\) represents the height, in feet, of the ball above the ground at time \(t\). What is the maximum height of the ball? At what time will the ball hit the ground?

- With a graph, we can see the number of seconds it takes for the ball to reach its peak, and also how long it takes to hit the ground. How can factoring the expression help us graph this function?

\[
\text{Change the expression to factored form. First, factor out the } -16 \text{ (GCF): } -16(t^2 - 9t - 10). \text{ Then, we can see that the quadratic expression remaining is factorable: } -16(t + 1)(t - 10).
\]
A.CED.A.1

A. Create equations that describe numbers or relationships
Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.

This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.

| Explanation: |
| This standard is taught in Algebra I and Algebra II. In Algebra I, tasks are limited to linear, quadratic, or exponential equations with integer exponents. Students recognize when a problem can be modeled with an equation or inequality and are able to write the equation or inequality. Students create, select, and use graphical, tabular and/or algebraic representations to solve the problem. |

| Examples: (Refer to the examples in Unit 1 in addition to the ones below.) |
| Solve for $d$: $3d^2 + d - 10 = 0$ |

A picture has a height that is $\frac{4}{3}$ its width. It is to be enlarged so that the ratio of height to width remains the same but the area is 192 sq. in. What are the dimensions of the enlargement?

Let $4x$ to $3x$ represent the ratio of height to width, $A = (h)(w)$, so we have

\[
\frac{4x}{3x} = 192 \\
12x^2 = 192 \\
x = 4 \text{ or } -4
\]

which means that dimensions are $h = 16$ and $w = 12$ because only positive values make sense in the context of area.

Eureka Math: Module 4 Lesson 1 - 2 Module 4 Lesson 5 Module 4 Lesson 7

This standard is revisited in Unit 5.
| 8.G.B.6 | **B. Understand and apply the Pythagorean Theorem**  
Explain a proof of the Pythagorean Theorem and its converse.  
8.MP.3. Construct viable arguments and critique the reasoning of others.  
8.MP.6. Attend to precision.  
8.MP.7. Look for and make use of structure. | **Explanation:**  
Using models, students explain the Pythagorean Theorem, understanding that the sum of the squares of the legs is equal to the square of the hypotenuse in a right triangle. Students also explain the converse of the Pythagorean Theorem. If the sum of the squares of two sides of a triangle equals the square of the third side, then the triangle is a right triangle. Students should be familiar with the common Pythagorean triplets.  
Below are a few illustrations used to prove the Pythagorean Theorem. The actual proofs can be found in the Eureka Math resource. Students should be able to explain several proofs of the theorem.  
1) “square within a square” proof (uses congruent triangles)  
   Module 2 Lesson 15  | **8th Gr. Eureka Math:**  
Module 2 Lesson 15  
Module 3 Lesson 13  
Module 7 Lesson 15  
Module 7 Lesson 15’s problem set is excellent to use for practice on this standard.  
**8th Gr. Big Ideas:**  
Sections: 7.3, 7.5 |
2) This proof uses similarity (Module 3 Lesson 13 and/or Module 7 Lesson 15)

3) “similar figures drawn from each side” (Module 7 Lesson 15)
4) Geometric illustration of proof (Module 7 Lesson 15)
The video located at the following link is an animation\(^1\) of the preceding proofs:  http://www.youtube.com/watch?v=QCyvxYLFSfU

**Examples:**

- For the right triangle shown below, identify and use similar triangles to illustrate the Pythagorean Theorem.

![Diagram of a right triangle with sides labeled a, b, and c.](image)

*Solution:* Module 7 Lesson 15 of the Eureka Math resource.

- After learning the proof of the Pythagorean Theorem using areas of squares, Joseph got really excited and tried explaining

\(^1\) Animation developed by Larry Francis.
it to his younger brother. He realized during his explanation that he had done something wrong. Help Joseph find his error. Explain what he did wrong.

Solution:
Based on the proof shown in class, we would expect the sum of the two smaller areas to be equal to the sum of the larger area, i.e., 16 + 25 should equal 49. However, 16 + 25 = 41. Joseph correctly calculated the areas of each square, so that was not his mistake. His mistake was claiming that a triangle with sides lengths of 4, 5, and 7 was a right triangle. We know that the Pythagorean Theorem only works for right triangles. Considering the converse of the Pythagorean Theorem, when we use the given side lengths, we do not get a true statement.

\[ 4^2 + 5^2 = 7^2 \]
\[ 16 + 25 = 49 \]
\[ 41 \neq 49 \]

Therefore, the triangle Joseph began with is not a right triangle, so it makes sense that the areas of the squares were not adding up like we expected.

- Explain a proof of the Pythagorean Theorem in your own words. Use diagrams and concrete examples, as necessary, to support your explanation.

Solutions will vary.

- The distance from Jonestown to Maryville is 180 miles, the distance from Maryville to Elm City is 300 miles, and the distance from Elm City to Jonestown is 240 miles. Do the three towns form a right triangle? Why or why not?

Solution:
If these three towns form a right triangle, then 300 would be the hypotenuse since it is the greatest distance.
\[ 180^2 + 240^2 = 300^2 \]
\[ 32400 + 57600 = 90000 \]
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<th>B. Understand and apply the Pythagorean Theorem</th>
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<td>Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.</td>
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<td>8.MP.1. Make sense of problems and persevere in solving them.</td>
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<td>8.MP.2. Reason abstractly and quantitatively.</td>
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<td>8.MP.5. Use appropriate tools strategically.</td>
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<td>8.MP.6. Attend to precision.</td>
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<tr>
<td>8.MP.7. Look for and make use of structure.</td>
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</tbody>
</table>

**Explanation:**
Through authentic experiences and exploration, students should use the Pythagorean Theorem to solve problems. Problems should include both mathematical and real-world contexts. Students should be familiar with the common Pythagorean triplets.

**Examples:**
- Use the Pythagorean Theorem to estimate the length of the unknown side of the right triangle. Explain why your estimate makes sense.

**Solution:**

![Right triangle with sides 6 cm and 11 cm]
Let $x$ be the length of the unknown side.

\[
6^2 + x^2 = 11^2 \\
36 + x^2 = 121 \\
x^2 = 85
\]

The length of unknown side of the triangle is approximately 9 cm. The number 85 is between the perfect squares 81 and 100. Since 85 is closer to 81 than 100, then the length of the unknown side of the triangle is closer to 9 than it is to 10.

- Determine the length of QS.

\[
Q \quad 17 \\
\quad 15 \\
\quad 25 \\
\quad T \\
R \\
S
\]

Solution:

\[
15^2 + |QT|^2 = 17^2 \\
225 + |QT|^2 = 289 \\
225 - 225 + |QT|^2 = 289 - 225 \\
|QT|^2 = 64 \\
|QT| = 8
\]

And
The Irrational Club wants to build a tree house. They have a 9-foot ladder that must be propped diagonally against the tree. If the base of the ladder is 5 feet from the bottom of the tree, how high will the tree house be off the ground?

Solution:

\[ 15^2 + |TS|^2 = 25^2 \]
\[ 225 + |TS|^2 = 625 \]
\[ 225 - 225 + |TS|^2 = 625 - 225 \]
\[ |TS|^2 = 400 \]
\[ |TS| = 20 \]

Since \(|QT| + |TS| = |QS|\), then the length of QS is \(8 + 20\), which is 28.

- The Irrational Club wants to build a tree house. They have a 9-foot ladder that must be propped diagonally against the tree. If the base of the ladder is 5 feet from the bottom of the tree, how high will the tree house be off the ground?

- The area of the right triangle below is 26.46 in\(^2\). What is the perimeter of the right triangle? Round your answer to the nearest tenths place.

Solution:

Let \(b\) equal the base of the triangle where \(h = 6.3\)
Let \( c \) equal the length of the hypotenuse.

\[
\begin{align*}
A &= \frac{bh}{2} \\
26.46 &= \frac{6.3b}{2} \\
52.92 &= 6.3b \\
52.92 &= 6.3b \\
6.3 &= 6.3 \\
8.4 &= b
\end{align*}
\]

The number \( \sqrt{110.25} \) is between 10 and 11. When comparing with tenths, the number is actually equal to 10.5 because \( 10.5^2 = 110.25 \). Therefore, the length of the hypotenuse is 10.5 inches. The perimeter of the triangle is \( 6.3 + 8.4 + 10.5 = 25.2 \) inches.

**8.G.B.8**

B. **Understand and apply the Pythagorean Theorem**

Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

- **8.MP.1.** Make sense of problems and persevere in solving them.
- **8.MP.2.** Reason abstractly and quantitatively.
- **8.MP.4.** Model with mathematics.
- **8.MP.5.** Use appropriate tools strategically.
- **8.MP.6.** Attend to precision.
- **8.MP.7.** Look for and make use of structure.

**Explanation:**

One application of the Pythagorean Theorem is finding the distance between two points on the coordinate plane. Students build on work from 6th grade (finding vertical and horizontal distances on the coordinate plane) to determine the lengths of the legs of the right triangle drawn connecting the points. Students understand that the line segment between the two points is the length of the hypotenuse.

Students find area and perimeter of two-dimensional figures on the coordinate plane, finding the distance between each segment of the figure. (Limit one diagonal line, such as a right trapezoid or parallelogram)

**Note:** The use of the distance formula is not an expectation in 8th grade.

**8th Gr. Eureka Math:**

Module 7 Lesson 17-18

**8th Gr. Big Ideas:**

Sections: 7.3, 7.5
Examples:

- **Example 1:**
  Find the length of \( AB \).

\[
\begin{align*}
6^2 + 7^2 &= c^2 \\
36 + 49 &= c^2 \\
85 &= c^2
\end{align*}
\]

**Solution:**
1. Form a right triangle so that the given line segment is the hypotenuse.
2. Use Pythagorean Theorem to find the distance (length) between the two points.

• Find the distance between \((-2, 4)\) and \((-5, -6)\).

**Solution:**
The distance between -2 and -5 is the horizontal length; the distance between 4 and -6 is the vertical distance.

○ Horizontal length: 3
Is the triangle formed by the following points a right triangle?

- A (1,1)
- B (11,1)
- C (2,4)

**Solution:**
Let $c$ represent the measure of $AC$.

$1^2 + 3^2 = c^2$
$1 + 9 = c^2$
$10 = c^2$
$c = \sqrt{10}$

Let $d$ represent the measure of $BC$.

$9^2 + 3^2 = d^2$
$81 + 9 = d^2$
$90 = d^2$
$d = \sqrt{90}$

The length of $AB$ is 10. $AB$ is the longest side. Using the Pythagorean Theorem:

$\sqrt{10^2} + \sqrt{90^2} = 10^2$
$10 + 90 = 100$
$100 = 100$

Therefore, the points A, B, and C form a right triangle.
Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.

This standard is revisited in Unit 5.

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<th>A.REI.B.4b</th>
<th>B. Solve equations and inequalities in one variable</th>
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<td>Solve quadratic equations in one variable.</td>
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<tr>
<td>b. Solve quadratic equations by inspection (e.g., for ( x^2 = 49 ), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form</td>
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**Explanation:**

Part b of this standard is taught in Algebra I and Algebra II. In Algebra I, tasks do not require students to write solutions for quadratic equations that have roots with nonzero imaginary parts. However, tasks can require that students recognize cases in which a quadratic equation has no real solutions.

Students should solve by factoring, completing the square, and using

**Examples:**

- A picture has a height that is \( \frac{4}{3} \) its width. It is to be enlarged so that the ratio of height to width remains the same but the area is 192 sq. in. What are the dimensions of the enlargement?

  Let \( 4x \) to \( 3x \) represent the ratio of height to width. \( A = (h)(w) \), so we have

  \[
  \frac{4x}{3x} = 192
  \]

  \[
  4x = 192 \\
  x = 48
  \]

- Find two consecutive odd integers whose product is 99. (Note: There are two different pairs of consecutive odd integers and only an algebraic solution will be accepted.)

  Let \( n \) represent the first odd integer and \( n + 2 \) represent the subsequent odd integer. The product is \( n(n + 2) \), which must equal 99. So, \( n^2 + 2n - 99 = 0 \).\n
  \[
  (n - 9)(n + 11) = 0 \\
  n = 9 \text{ or } n = -11
  \]

  If \( n = 9 \), then \( n + 2 = 11 \), so the numbers could be 9 and 11. Or if \( n = -11 \), then \( n + 2 = -9 \), so the numbers could be \(-11\) and \(-9\).
of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers $a$ and $b$.

the quadratic formula. The zero product property is used to explain why the factors are set equal to zero. Students should relate the value of the discriminant to the type of root to expect. A natural extension would be to relate the type of solutions to $ax^2 + bx + c = 0$ to the behavior of the graph of $y = ax^2 + bx + c$.

<table>
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<th>Value of Discriminant</th>
<th>Nature of Roots</th>
<th>Nature of Graph</th>
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<tr>
<td>$b^2 - 4ac = 0$</td>
<td>1 real roots</td>
<td>intersects x-axis once</td>
</tr>
<tr>
<td>$b^2 - 4ac &gt; 0$</td>
<td>2 real roots</td>
<td>intersects x-axis twice</td>
</tr>
<tr>
<td>$b^2 - 4ac &lt; 0$</td>
<td>2 complex roots</td>
<td>does not intersect x-axis</td>
</tr>
</tbody>
</table>

Examples:

- Are the roots of $2x^2 + 5 = 2x$ real or complex? How many roots does it have?

- What is the nature of the roots of $x^2 + 6x - 10 = 0$? Solve the equation using the quadratic formula and completing the square. How are the two methods related?

- Elegant ways to solve quadratic equations by factoring for those involving expressions of the form: $ax^2$ and $a(x-b)^2$

  $3x^2 - 9 = 0$
  $3x^2 = 9 \Rightarrow x^2 = 3 \Rightarrow x = \pm \sqrt{3}$

  $(x - 3)^2 = 1$
  $(x - 3) = \pm 1 \Rightarrow x = 3 \pm 1 = 2$ or $4$
A.REI.D.11

D. Represent and solve equations and inequalities graphically

Explain why the $x$-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.

**Example:** Ryan used the quadratic formula to solve an equation and his result was $x = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$.

- a. Write the quadratic equation Ryan started with.
- b. Simplify the expression to find the solutions.
- c. What are the $x$-intercepts of the graph of the corresponding quadratic function?
- d. Factoring ($a = 1$)

<table>
<thead>
<tr>
<th>Step</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>a.</td>
<td>$x^2 = 49$</td>
</tr>
<tr>
<td>b.</td>
<td>$3x^2 + 9 = 72$</td>
</tr>
<tr>
<td>c.</td>
<td>$4x^2 + 13x - 7 = 0$</td>
</tr>
<tr>
<td>d.</td>
<td>$x^2 + 8x + 12 = 0$</td>
</tr>
</tbody>
</table>

**Explanation:**

This standard is taught in Algebra I and Algebra II. In Algebra I, tasks that assess conceptual understanding of the indicated concept may involve any of the function types mentioned in the standard except exponential and logarithmic functions. Finding the solutions approximately is limited to cases where $f(x)$ and $g(x)$ are polynomial functions.

Students need to understand that numerical solution methods (data in a table used to approximate an algebraic function) and graphical solution methods may produce approximate solutions, and algebraic solution methods produce precise solutions that can be represented graphically or numerically. Students may use graphing calculators or programs to generate tables of values, graph, or solve a variety of functions.

**Examples:** (Refer to examples in Unit 2 in addition to the example below)

- Find $P(0)$ and $R(0)$ values and explain what it means in the problem. What conclusion can you make based on these values? Did they throw the ball from the same place? Explain your answer.

$P(0) = 80$ and $R(0) = 86$. Students interpret the $y$-intercept in context. Pettitte and Ryu threw the ball 80 ft. and 86 ft. above the ground, respectively. They were throwing the ball from different places on top of a building.

Eureka Math:
Module 4 Lesson 10
<table>
<thead>
<tr>
<th>F.IF.B.4</th>
<th>B. Interpret functions that arise in applications in terms of the context</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.</td>
</tr>
<tr>
<td></td>
<td>This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.</td>
</tr>
<tr>
<td></td>
<td>Explanation: This standard is taught in Algebra I and Algebra II. In Algebra I, tasks have a real-world context and they are limited to linear functions, quadratic functions, square-root functions, cube-root functions, piecewise functions (including step functions and absolute-value functions), and exponential functions with domains in the integers. Some functions “tell a story” hence the portion of the standard that has students sketching graphs given a verbal description. Students should have experience with a wide variety of these types of functions and be flexible in thinking about functions and key features using tables and graphs. Examples of these can be found at <a href="http://graphingstories.com">http://graphingstories.com</a></td>
</tr>
<tr>
<td></td>
<td>Students may be given graphs to interpret or produce graphs given an expression or table for the function, by hand or using technology.</td>
</tr>
<tr>
<td></td>
<td>Examples: (Refer to examples in Module 3 in addition to the examples below)</td>
</tr>
<tr>
<td></td>
<td>• Compare the graphs of ( y = 3x^2 ) and ( y = 3x^3 ).</td>
</tr>
<tr>
<td></td>
<td>• It started raining lightly at 5am, then the rainfall became heavier at 7am. By 10am the storm was over, with a total rainfall of 3 inches. It didn’t rain for the rest of the day. Sketch a possible graph for the number of inches of rain as a function of time, from midnight to midday.</td>
</tr>
<tr>
<td></td>
<td>Eureka Math: Module 4 Lesson 8 Module 4 Lesson 10 This standard is revisited in Unit 5.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>F.IF.B.5</th>
<th>B. Interpret functions that arise in applications in terms of the context</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function ( h(n) ) gives the number of person-hours it takes to assemble ( n ) engines in a factory, then the positive integers would be an appropriate domain for the function.</td>
</tr>
<tr>
<td></td>
<td>This is a modeling standard which means students choose and use appropriate mathematics to analyze</td>
</tr>
<tr>
<td></td>
<td>Explanation: Students explain the domain of a function from a given context. Students may explain orally, or in written format, the existing relationships. Given the graph of a function, determine the practical domain of the function as it relates to the numerical relationship it describes.</td>
</tr>
<tr>
<td></td>
<td>Examples: (Refer to the examples in Unit 2)</td>
</tr>
<tr>
<td></td>
<td>Eureka Math: Module 4 Lesson 7-10 This standard is revisited in Unit 5.</td>
</tr>
<tr>
<td><strong>F.IF.B.6</strong></td>
<td>B. Interpret functions that arise in applications in terms of the context</td>
</tr>
<tr>
<td>--------------</td>
<td>--------------------------------------------------------------------------</td>
</tr>
<tr>
<td></td>
<td>Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.</td>
</tr>
<tr>
<td></td>
<td>This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.</td>
</tr>
<tr>
<td></td>
<td>Explanation: Students were first introduced to the concept of rate of change in grade 6 and continued exploration of the concept throughout grades 7 and 8. In Algebra I, students will extend this knowledge to non-linear functions.</td>
</tr>
<tr>
<td></td>
<td>This standard is taught in Algebra I and Algebra II. In Algebra I, tasks have a real-world context and are limited to linear functions, quadratic functions, square root functions, cube root functions, piecewise-defined functions (including step functions and absolute value functions), and exponential functions with domains in the integers.</td>
</tr>
<tr>
<td></td>
<td>Examples: (Refer to the examples in Unit 2)</td>
</tr>
<tr>
<td></td>
<td>Eureka Math: Module 4 Lesson 10</td>
</tr>
<tr>
<td></td>
<td>This standard is revisited in Unit 5.</td>
</tr>
<tr>
<td><strong>F.IF.C.7ab</strong></td>
<td>C. Analyze functions using different representations</td>
</tr>
<tr>
<td></td>
<td>Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.</td>
</tr>
<tr>
<td></td>
<td>a. Graph linear and quadratic functions and show intercepts, maxima, and minima.</td>
</tr>
<tr>
<td></td>
<td>b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.</td>
</tr>
<tr>
<td></td>
<td>This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.</td>
</tr>
<tr>
<td></td>
<td>Explanation: This standard was introduced in Unit 2 with the focus on linear functions, piecewise functions (including step functions and absolute-value functions), and exponential functions with domains in the integers.</td>
</tr>
<tr>
<td></td>
<td>In this module, the focus is on quadratic functions, square root functions and cube root functions. Key characteristics include but are not limited to maxima, minima, intercepts, symmetry, and end behavior. Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to graph functions.</td>
</tr>
<tr>
<td></td>
<td>Examples: (Refer to examples in Unit 2 in addition to the examples below)</td>
</tr>
<tr>
<td></td>
<td>Eureka Math: Module 4 Lesson 8 Module 4 Lesson 9</td>
</tr>
</tbody>
</table>
Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. This module alternates between Eureka Math: Module 4 Lesson 6.
algebraic manipulation of expressions and equations and interpretation of the quantities in the relationship in terms of the context. Students must be able to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own without necessarily attending to their referents, and then to contextualize—to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning requires the habit of creating a coherent representation of the problem at hand, considering the units involved, attending to the meaning of quantities (not just how to compute them), knowing different properties of operations, and flexibility in using them.

<table>
<thead>
<tr>
<th>MP.3</th>
<th>Construct viable arguments and critique the reasoning of others.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>MP.4</th>
<th>Model with mathematics.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In this module, students create a function from a contextual situation described verbally, create a graph of their function, interpret key features of both the function and the graph (in the terms of the context), and answer questions related to the function and its graph. They also create a function from a data set based on a contextual situation. Lesson 1 asks students to use geometric models to demonstrate their understanding of multiplication of polynomials. Lesson 2 students represent multiplication of binomials and factoring</td>
</tr>
</tbody>
</table>
| MP.7 | Look for and make use of structure. | Mathematically proficient students look closely to discern a pattern or structure. They can see algebraic expressions as single objects, or as a composition of several objects. In this Module, students use the structure of expressions to find ways to rewrite them in different but equivalent forms. For example, in the expression $x^2 + 9x + 14$, students must see the 14 as $2 \times 7$ and the 9 as $2 + 7$ to find the factors of the quadratic. In relating an equation to a graph, they can see $y = -3(x - 1)^2 + 5$ as 5 added to a negative number times a square and realize that its value cannot be more than 5 for any real domain value.

Throughout lesson 3, students are asked to make use of the structure of an expression, seeing part of a complicated expression as a single entity in order to factor quadratic expressions and to compare the areas in using geometric and tabular models.

In lesson 4, students look to discern a pattern or structure in order to rewrite a quadratic trinomial in an equivalent form. | Eureka Math: Module 4 Lesson 3 Module 4 Lesson 4 Module 4 Lesson 6 |
In Topic B, students apply their experiences from Topic A as they transform standard form quadratic functions into the completed square form \((x) = a(x - h)^2 + k\) (sometimes referred to as the vertex form). Known as, completing the square, this strategy is used to solve quadratic equations when the quadratic expression cannot be factored (A-SSE.B.3b). Students recognize that this form reveals specific features of quadratic functions and their graphs, namely the minimum or maximum of the function (the vertex of the graph) and the line of symmetry of the graph (A-APR.B.3, F-IF.B.4, F-IF.C.7a). Students derive the quadratic formula by completing the square for a general quadratic equation in standard form \((y = ax^2 + bx + c)\) and use it to determine the nature and number of solutions for equations when \(y\) equals zero (A-SSE.A.2, A-REI.B.4). For quadratics with irrational roots students use the quadratic formula and explore the properties of irrational numbers (N-RN.B.3). With the added technique of completing the square in their toolboxes, students come to see the structure of the equations in their various forms as useful for gaining insight into the features of the graphs of equations (A-SSE.B.3). Students study business applications of quadratic functions as they create quadratic equations and/or graphs from tables and contexts and use them to solve problems involving profit, loss, revenue, cost, etc. (A-CED.A.1, A-CED.A.2, F-IF.B.6, F-IF.C.8a). In addition to applications in business, they also solve physics-based problems involving objects in motion. In doing so, students also interpret expressions and parts of expressions, in context and recognize when a single entity of an expression is dependent or independent of a given quantity (A-SSE.A.1).

| Big Idea: | The vertex of a quadratic function provides the maximum or minimum output value of the function and the input at which it occurs. Every quadratic equation can be solved using the Quadratic Formula. |
| Essential Questions: | How is the quadratic formula related to completing the square? |
| Vocabulary | Complete the square, Business application: unit price, quantity, revenue, unit cost, profit, standard form of a quadratic function, vertex, quadratic formula |
| Assessment | Galileo: Topic B Assessment |

<table>
<thead>
<tr>
<th>Standard</th>
<th>AZ College and Career Readiness Standards</th>
<th>Explanations &amp; Examples</th>
<th>Resources</th>
</tr>
</thead>
<tbody>
<tr>
<td>N.RN.B.3</td>
<td>B. Use properties of rational and irrational numbers.</td>
<td>Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.</td>
<td>Eureka Math: Module 4 Lesson 13</td>
</tr>
</tbody>
</table>

**Explanations & Examples:**

- The foundation for this standard was taught in grades 6-8 with students understanding rational and irrational numbers.

Since every difference is a sum and every quotient is a product, this includes differences and quotients as well. Explaining why the four operations on rational numbers produce rational numbers can be a review of students understanding of fractions and negative numbers. Explaining why the sum of a rational and an irrational number is irrational, or why the product is irrational, includes reasoning about the inverse relationship between addition and subtraction (or between
Students know and justify that when

- adding or multiplying two rational numbers the result is a rational number.
- adding a rational number and an irrational number the result is irrational.
- multiplying of a nonzero rational number and an irrational number the result is irrational.

Examples:

- Explain why the number $2\pi$ must be irrational, given that $\pi$ is irrational.

  **Sample Response:** If $2\pi$ were rational, then half of $2\pi$ would also be rational, so $\pi$ would have to be rational as well.

A.SSE.A.1 A. Interpret the structure of expressions
Interpret expressions that represent a quantity in terms of its context

- Interpret parts of an expression, such as terms, factors, and coefficients.
- Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret $P(1+r)^n$ as the product of $P$ and a factor not depending on $P$.

  *This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.*

Explanation:
This standard is taught in Algebra I and Algebra II. In Algebra I the focus is on linear expressions, exponential expressions with integer exponents and quadratic expressions. Throughout Algebra I, students should:

- **Explain** the difference between an expression and an equation.
- **Use** appropriate vocabulary for the parts that make up the whole expression.
- **Identify** the different parts of the expression and explain their meaning within the context of the problem.
- **Decompose** expressions and make sense of the multiple factors and terms by explaining the meaning of the individual parts.

**Note:** Students should understand the vocabulary for the parts that make up the whole expression, be able to identify those parts, and interpret their meaning in terms of a context.

Eureka Math: Module 4 Lesson 11 - 17
<table>
<thead>
<tr>
<th>A.SSE.A.2</th>
<th>A. Interpret the structure of expressions</th>
<th>Explanation: This standard is taught in Algebra I and Algebra II. In Algebra I tasks are limited to numerical and polynomial expressions in one variable, with a focus on quadratics. Examples: Recognize that $53^2 - 47^2$ is the difference of squares and see an opportunity to rewrite it in the easier-to-evaluate form $(53 - 47)(53 + 47)$. See an opportunity to rewrite $a^2 + 9a + 14$ as $(a + 7)(a + 2)$. Can include the sum or difference of cubes (in one variable), and factoring by grouping.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Use the structure of an expression to identify ways to rewrite it. <em>For example, see</em> $x^2 - y^2$ as $(x^2)^2 - (y^2)^2$, <em>thus recognizing it as a difference of squares that can be factored as</em> $(x^2 - y^2)(x^2 + y^2)$.</td>
<td>Use factoring techniques such as common factors, grouping, the difference of two squares, or a combination of methods to factor quadratics completely. Students should extract the greatest common factor (whether a constant, a variable or a combination of each). If the remaining expressions is a factorable quadratic, students should factor the expression further.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>If the leading coefficient for a quadratic expression is not 1, the first step in factoring should be to see if all the terms in the expanded form have a common factor. Then after factoring out the greatest common factor, it may be possible to factor again.</td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>Examples:</strong></td>
</tr>
<tr>
<td></td>
<td></td>
<td><em>Factor</em> $2x^3 - 50x$ completely:</td>
</tr>
<tr>
<td></td>
<td></td>
<td>The GCF of the expression is $2x$, $2x(x^2 - 25)$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Now factor the difference of squares: $2x(x - 5)(x + 5)$</td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>Factor completely:</strong> $3t^3 + 18t^2 - 48t$</td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>Factor completely:</strong> $4n - n^3$</td>
</tr>
</tbody>
</table>

**Eureka Math:**
Module 4 Lesson 11
Module 4 Lesson 12
Module 4 Lesson 13
Module 4 Lesson 14
A.SSE.B.3b  

B. Write expressions in equivalent forms to solve problems

Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.

This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.

### Examples:

- The quadratic expression \(-x^2 - 24x + 55\) models the height of a ball thrown vertically. Identify the vertex-form of the expression, determine the vertex from the rewritten form, and interpret its meaning in this context.

- \(16a^2b^4 + 20ab^2 - 6\)
  \((4ab^2)^2 + 5(4ab^2) - 6\)
  \((4ab^2 + 6)(4ab^2 - 1)\)

  **Factored form:** \(2(2ab^2 + 3)(4ab^2 - 1)\)

### Table:

<table>
<thead>
<tr>
<th>Standard Form</th>
<th>Factored Form</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x^2 + 12x + 36)</td>
<td>((x + 6)^2)</td>
</tr>
<tr>
<td>(x^2 - 12x + 36)</td>
<td>((x - 6)^2)</td>
</tr>
<tr>
<td>(x^2 + 20x + 100)</td>
<td>((x + 10)^2)</td>
</tr>
<tr>
<td>(x^2 - 3x + \frac{9}{4})</td>
<td>((x - \frac{3}{2})^2)</td>
</tr>
<tr>
<td>(x^2 + 100x + 2,500)</td>
<td>((x + 50)^2)</td>
</tr>
</tbody>
</table>

### Explanation:

Write expressions in equivalent forms by completing the square to convey the vertex form, to find the maximum or minimum value of a quadratic function, and to explain the meaning of the vertex.

- The quadratic expression \(-x^2 - 24x + 55\) models the height of a ball thrown vertically. Identify the vertex-form of the expression, determine the vertex from the rewritten form, and interpret its meaning in this context.

- \(3x^2 + 12x - 8\)
  \(3(x^2 + 4x) - 8 - 3(x + 2)^2 - 8 - 12 - 3(x + 2)^2 - 20\)

- \(4p^2 - 12p + 13\)
  \(4(p^2 - 3p) + 13 - 4\left(\frac{3}{2}\right)^2 + 13 - 9 - 4\left(\frac{3}{2}\right)^2 + 4\)

Eureka Math: Module 4 Lesson 11-14

6/12/2016
Rewrite each expression by completing the square.

1. \( a^2 - 4a + 15 \)
   
   \[(a - 2)^2 + 11\]  \(\text{Note: Since the constant term required to complete the square is less than the constant term, +15, students may notice that they just need to split the +15 strategically.}\

2. \( n^2 - 2n - 15 \)
   
   \[(n - 1)^2 - 16\]

3. \( c^2 + 20c - 40 \)
   
   \[(c + 10)^2 - 140\]

A fast food restaurant has determined that their price function is \( P = \frac{x}{20,000} \) where \( x \) represents the number of hamburgers sold.

a. The cost of producing \( x \) hamburgers is determined by the expression \( 5,000 + 0.56x \). Write an expression representing the profit for selling \( x \) hamburgers.

\[
\text{Profit} = \text{Revenue} - \text{Costs} = (\text{quantity})(\text{price}) - \text{cost} = (x) \left( 3 - \frac{x}{20,000} \right) - (5,000 + 0.56x)
\]

\[
= 3x - \frac{x^2}{20,000} - 5,000 - 0.56x = -\frac{x^2}{20,000} + 2.44x - 5,000
\]

b. Complete the square for your expression in part (a) to determine the number of hamburgers that need to be sold to maximize the profit, given this price function.

\[
-\frac{1}{20,000} (x^2 - 48,000x + \_ \_ \_ \_ \_ \_ \_ ) - 5,000 = -\frac{1}{20,000} (x^2 - 48,000x + 24,000^2) - 5,000 + \frac{24,400^2}{20,000}
\]

\[
-\frac{1}{20,000} (x - 24,400)^2 - 5,000 + 29,768 = -\frac{1}{20,000} (x - 24,400)^2 + 24,768
\]

So, 24,400 hamburgers must be sold to maximize the profit using this price expression. \( \text{Note: The profit will be 224,768.} \)
B. Solve equations and inequalities in one variable

Solve quadratic equations in one variable.

a. Use the method of completing the square to transform any quadratic equation in \(x\) into an equation of the form \((x - p)^2 = q\) that has the same solutions. Derive the quadratic formula from this form.

b. Solve quadratic equations by inspection (e.g., for \(x = 49\)), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as \(a \pm bi\) for real numbers \(a\) and \(b\).

**Explanation:**

Part b of this standard is taught in Algebra I and Algebra II. In Algebra I, tasks do not require students to write solutions for quadratic equations that have roots with nonzero imaginary parts. However, tasks can require that students recognize cases in which a quadratic equation has no real solutions.

Students should solve by factoring, completing the square, and using the quadratic formula. The zero product property is used to explain why the factors are set equal to zero. Students should relate the value of the discriminant to the type of root to expect. A natural extension would be to relate the type of solutions to \(ax^2 + bx + c = 0\) to the behavior of the graph of \(y = ax^2 + bx + c\).

<table>
<thead>
<tr>
<th>Value of Discriminant</th>
<th>Nature of Roots</th>
<th>Nature of Graph</th>
</tr>
</thead>
<tbody>
<tr>
<td>(b^2 - 4ac = 0)</td>
<td>1 real roots</td>
<td>intersects x-axis once</td>
</tr>
<tr>
<td>(b^2 - 4ac &gt; 0)</td>
<td>2 real roots</td>
<td>intersects x-axis twice</td>
</tr>
<tr>
<td>(b^2 - 4ac &lt; 0)</td>
<td>2 complex roots</td>
<td>does not intersect x-axis</td>
</tr>
</tbody>
</table>

**Examples Part a:**

*Example:* Solve \(-2x^2 - 16x = 20\)

Students who writes vertex form first

\[
-2x^2 - 16x - 20 = 0 \\
-2(x^2 + 8x + 16) = 20 - 32 = 0 \\
-2(x - 4)^2 = 0 \\
(x - 4)^2 = 0 \\
(x - 4) = 0 \\
x = 4 \\
\]

Student who adapts method

\[
-2(x^2 - 8x) = 20 \\
-2(x^2 - 8x + 16) = 20 + 32 \\
-2(x - 4)^2 = 52 \\
(x - 4)^2 = 26 \\
x - 4 = \pm \sqrt{26} \\
x = 4 \pm \sqrt{26} \\
\]

**Examples Part b:**

- Are the roots of \(2x^2 + 5 = 2x\) real or complex? How many roots does it have?
- What is the nature of the roots of \(x^2 + 6x - 10 = 0\)? Solve the equation using the quadratic formula and completing the square. How are the two methods related?
- Solve:
B. Understand the relationship between zeros and factors of polynomials

Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

Example:

\[
\begin{align*}
x^2 - 2x &= 12 \\
x^2 - 2x + 1 &= 12 + 1 \\
(x - 1)^2 &= 13 \\
&= 1 \pm \sqrt{13}
\end{align*}
\]

\[
\begin{align*}
\frac{1}{2}r^2 - 6r &= 2 \\
\frac{1}{2}(r^2 - 12r + 36) &= 2 + 18 \\
\frac{1}{2}(r - 6)^2 &= 20 \\
(r - 6)^2 &= 40 \\
r - 6 &= \pm \sqrt{40} \\
r &= 6 \pm \sqrt{40} = 6 \pm 2\sqrt{10}
\end{align*}
\]

[Be careful with factoring out the rational leading coefficient.]

[The last step should be optional at this point.]

Use the quadratic formula to solve each equation.

1. \(x^2 - 2x = 12 \rightarrow a = 1, b = -2, c = -12\) (watch the negatives)

\[
x = \frac{2 \pm \sqrt{(-2)^2 - 4(1)(-12)}}{2(1)} = \frac{2 \pm \sqrt{52}}{2} = 1 \pm \sqrt{13}
\]

Explanation:

This standard is taught in Algebra I and Algebra II. In Algebra I, tasks are limited to quadratic and cubic polynomials, in which linear and quadratic factors are available. For example, find the zeros of \((x - 2)(x^2 - 9)\). Graphing calculators or programs can be used to generate graphs of polynomial functions.

Eureka Math:
Module 4 Lesson 14
A. Create equations that describe numbers or relationships

Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*

*This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.*

**Explanation:**

This standard is taught in Algebra I and Algebra II. In Algebra I, tasks are limited to linear, quadratic or exponential equations with integer exponents. Students recognize when a problem can be modeled with an equation or inequality and are able to write the equation or inequality. Students create, select, and use graphical, tabular and/or algebraic representations to solve the problem.

Equations can represent real world and mathematical problems. Include equations and inequalities that arise when comparing the values of two different functions, such as one describing linear growth and one describing exponential growth.

**Examples:**

- Lava coming from the eruption of a volcano follows a parabolic path. The height \( h \) in feet of a piece of lava \( t \) seconds after it leaves the crater is given by the equation:

\[
h(t) = -16t^2 + 144t + 160,
\]

where \( t \) represents the time the ball is in the air in seconds and \( h(t) \) represents the height, in feet, of the ball above the ground at time \( t \). What is the maximum height of the ball? At what time will the ball hit the ground?

a. With a graph, we can see the number of seconds it takes for the ball to reach its peak, and also how long it takes to hit the ground. How can factoring the expression help us graph this function?

Change the expression to factored form. \( \text{First, factor out the } -16 \text{ (GCF): } -16(t^2 - 9t - 10) \). Then, we can see that the quadratic expression remaining is factorable: \( -16(t + 1)(t - 10) \).

b. Once we have the function in its factored form what do we need to know in order to graph it? Now graph the function.

We can find the \( t \) intercepts, \( y \) intercept, axis of symmetry, and the vertex then sketch the graph of the function. \( t \) intercepts are \( (10, 0) \) and \( (-1, 0) \); \( y \) intercept is \( (0, 160) \); the axis of symmetry is \( t = 4.5 \); and the vertex is \( (4.5, 484) \). (Note: We find the \( y \) coordinate of the vertex by substituting 4.5 into each form of the equation.)
<table>
<thead>
<tr>
<th>A.CED.A.2</th>
<th>A. Create equations that describe numbers or relationships</th>
</tr>
</thead>
<tbody>
<tr>
<td>Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. <strong>This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.</strong></td>
<td></td>
</tr>
</tbody>
</table>

**Explanation:**  
This standard is taught in Algebra I and Algebra II. In Algebra I, students create equations in two variables for linear, exponential and quadratic contextual situations. Limit exponential situations to only ones involving integer input values. The focus in this Unit is on quadratics.  

**Examples:** (Refer to examples from Unit 1 and Unit 3 Topic A)  
- Caitlin has 60 feet of material that can be used to make a fence. Using this material, she wants to create a rectangular pen for her dogs to play in. What dimensions will maximize the area of the pen?  
  a. Let $w$ be the width of the rectangular pen in feet. Write an expression that represents the length when the width is $w$ feet.  
  $$
  \frac{60-2w}{2} \text{ or } 30 - w
  $$  
  b. Define a function $A(w)$ that describes the area, $A$, in terms of the width, $w$.  
  $$
  A(w) = w(30 - w) \text{ or } A(w) = -w^2 + 30w
  $$  
  Write two different equations representing quadratic functions whose graphs have vertices at (4.5, -8).  

<table>
<thead>
<tr>
<th>F.IF.B.4</th>
<th>B. Interpret functions that arise in applications in terms of the context</th>
</tr>
</thead>
</table>
| For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and **periodicity.**  

**Explanation:**  
This standard is taught in Algebra I and Algebra II. In Algebra I, tasks have a real-world context and they are limited to linear functions, quadratic functions, square-root functions, cube-root functions, piecewise functions (including step functions and absolute-value functions), and exponential functions with domains in the integers. Some functions "tell a story" hence the portion of the standard that has students sketching graphs given a verbal description. Students should have experience with a wide variety of these types of functions and be flexible in thinking about functions and key features using tables and graphs. Examples of these can be found at [http://graphingstories.com](http://graphingstories.com)  

**Eureka Math:**  
Module 4 Lesson 16, 17  
This standard is revisited in Unit 5.
<table>
<thead>
<tr>
<th><strong>F.IF.B.6</strong></th>
<th><strong>B. Interpret functions that arise in applications in terms of the context</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Explanation:</strong> Students were first introduced to the concept of rate of change in grade 6 and continued exploration of the concept throughout grades 7 and 8. In Algebra I, students will extend this knowledge to non-linear functions.</td>
<td></td>
</tr>
<tr>
<td><strong>Examples:</strong> (Refer to examples in Units 2 and 3)</td>
<td></td>
</tr>
<tr>
<td><strong>Eureka Math:</strong> Module 4 Lesson 17</td>
<td></td>
</tr>
<tr>
<td>This standard is revisited in Unit 5.</td>
<td></td>
</tr>
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</table>

<table>
<thead>
<tr>
<th><strong>F.IF.C.7ab</strong></th>
<th><strong>C. Analyze functions using different representations</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Explanation:</strong> This standard was introduced in Unit 2 with the focus on linear functions, piecewise functions (including step functions and absolute-value functions), and exponential functions with domains in the integers.</td>
<td></td>
</tr>
<tr>
<td><strong>Examples:</strong> (Refer to examples in Unit 2)</td>
<td></td>
</tr>
</tbody>
</table>
| **Eureka Math:** Module 4 Lesson 16
Module 4 Lesson 17 |
<table>
<thead>
<tr>
<th>F.IF.C.8a</th>
<th>C. Analyze functions using different representations</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.</td>
</tr>
<tr>
<td></td>
<td>a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.</td>
</tr>
</tbody>
</table>

**Explanation:**
Students must use the factors to reveal and explain properties of the function, interpreting them in context. Factoring just to factor does not fully address this standard.

**Examples:**
- The quadratic expression $-5x^2 + 10x + 15$ represents the height of a diver jumping into a pool off a platform. Use the process of factoring to determine key properties of the expression and interpret them in the context of the problem.
- Caflin has 60 feet of material that can be used to make a fence. Using this material, she wants to create a rectangular pen for her dogs to play in. What dimensions will maximize the area of the pen?
  a. Let $w$ be the width of the rectangular pen in feet. Write an expression that represents the length when the width is $w$ feet.
    
    $\frac{(60-2w)}{2} \text{ or } 30-w$
  
    b. Define a function $A(w)$ that describes the area, $A$, in terms of the width, $w$.
    
    $A(w) = w(30 - w)$ or $A(w) = -w^2 + 30w$
  
    c. Rewrite $A(w)$ in vertex form.
    
    $A(w) = -(w^2 - 30w)$
    
    $= -(w^2 - 30w + 225) + 225$
    
    $= -(w - 15)^2 + 225$
| MP.1 | Make sense of problems and persevere in solving them. | Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. In Module 4, students make sense of problems by analyzing the critical components of the problem, a verbal description, data set, or graph and persevere in writing the appropriate function to describe the relationship between two quantities. | Eureka Math: Module 4 Lesson 13  Module 4 Lesson 14 |
| MP.2 | Reason abstractly and quantitatively. | Mathematically proficient students make sense of quantities and their relationships in problem situations. This module alternates between algebraic manipulation of expressions and equations and interpretation of the quantities in the relationship in terms of the context. Students must be able to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own without necessarily attending to their referents, and then to contextualize—to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning requires the habit of creating a coherent representation of the problem at hand, considering the units involved, attending to the meaning of quantities (not just how to compute them), knowing different properties of operations, and flexibility in using them. | Eureka Math: Module 4 Lesson 17 |
| MP.4 | Model with mathematics. | Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In this module, students create a function from a contextual situation described verbally, create a graph of their function, interpret key features of both the function and the graph (in the terms of the context), and answer questions related to the function. | Eureka Math: Module 4 Lesson 16 |
They also create a function from a data set based on a contextual situation. In Topic C, students use the full modeling cycle. They model quadratic functions presented mathematically or in a context. They explain the reasoning used in their writing or using appropriate tools, such as graphing paper, graphing calculator, or computer software.

<table>
<thead>
<tr>
<th>MP.7</th>
<th>Look for and make use of structure.</th>
</tr>
</thead>
</table>
| Mathematically proficient students look closely to discern a pattern or structure. They can see algebraic expressions as single objects, or as a composition of several objects. In this Module, students use the structure of expressions to find ways to rewrite them in different but equivalent forms. For example, in the expression $$x^2 + 9x + 14$$, students must see the 14 as $$2 \times 7$$ and the 9 as $$2 + 7$$ to find the factors of the quadratic. In relating an equation to a graph, they can see $$y = -3(x - 1)^2 + 5$$ as 5 added to a negative number times a square and realize that its value cannot be more than 5 for any real domain value. | Eureka Math:  
Module 4 Lesson 11  
Module 4 Lesson 12  
Module 4 Lesson 14  
Module 4 Lesson 16 |

<table>
<thead>
<tr>
<th>MP.8</th>
<th>Look for and express regularity in repeated reasoning.</th>
</tr>
</thead>
</table>
| Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results. | Eureka Math:  
Module 4 Lesson 16 |
In topic C, students explore the families of functions that are related to the parent functions, specifically for quadratic \( f(x) = x^2 \), square root \( f(x) = \sqrt{x} \), and cube root \( f(x) = \sqrt[3]{x} \), to perform first horizontal and vertical translations and shrinking and stretching the functions (F-IF.C.7b, F-BF.B.3). They recognize the application of transformations in the vertex form for the quadratic function and use it to expand their ability to efficiently sketch graphs of square and cube root functions. Students compare quadratic, square root, or cube root functions in context, and each represented in different ways (verbally with a description, as a table of values, algebraically, or graphically). In the final two lessons, students are given real-world problems of quadratic relationships that may be given as a data set, a graph, described relationship, and/or an equation. They choose the most useful form for writing the function and apply the techniques learned throughout the module to analyze and solve a given problem (A-CED.A.2), including calculating and interpreting the rate of change for the function over an interval (F-IF.B.6).

**Big Idea:**
- The key features of a quadratic function, which are the zeros (roots), the vertex, and the leading coefficient, can be used to interpret the function in a context.

**Essential Questions:**
- What is the relevance of the vertex in physics and business applications?

**Vocabulary**
Horizontal/vertical stretch, negative scale factor, shrink, parent function, vertical scaling, scale factor,

**Assessment**
Galileo: Topic C Assessment

<table>
<thead>
<tr>
<th>Standard</th>
<th>AZ College and Career Readiness Standards</th>
<th>Explanations &amp; Examples</th>
<th>Resources</th>
</tr>
</thead>
</table>
| A.CED.A.2 | A. Create equations that describe numbers or relationships | Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.  
*This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.* | Explanation:  
This standard is taught in Algebra I and Algebra II. In Algebra I, students create equations in two variables for linear, exponential and quadratic contextual situations. Limit exponential situations to only ones involving integer input values. The focus in this unit is on quadratics.  

Examples: (Refer to examples from Unit 1 and Unit 3 Topics A and B in addition to the examples below)  
-  | Eureka Math:  
Module 4 Lesson 19, 21, 23, 24  
This standard is revisited in Unit 5. |
F.IF.B.6

B. Interpret functions that arise in applications in terms of the context

Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a

Explanation:
Students were first introduced to the concept of rate of change in grade 6 and continued exploration of the concept throughout grades 7 and 8. In Algebra I, students will extend this knowledge to non-linear functions.

This standard is revisited in Unit 5.
<table>
<thead>
<tr>
<th>F.IF.C.7ab</th>
<th>C. Analyze functions using different representations</th>
<th><strong>Explanation:</strong></th>
<th>Eureka Math: Module 4 Lesson 18 - 23</th>
</tr>
</thead>
</table>
| | Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.  
  a. Graph linear and quadratic functions and show intercepts, maxima, and minima.  
  b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.  
  *This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.* | This standard was introduced in Unit 2 with the focus on linear functions, piecewise functions (including step functions and absolute-value functions), and exponential functions with domains in the integers.  
In this module, the focus is on quadratic functions, square root functions and cube root functions. Key characteristics include but are not limited to maxima, minima, intercepts, symmetry, and end behavior. Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to graph functions.  
Examples: (Refer to examples in Unit 2) | |
| F.IF.C.8a | C. Analyze functions using different representations | **Examples:** (Refer to the examples in topic B in addition to the example below.) | Eureka Math: Module 4 Lesson 21  
Module 4 Lesson 22  
Module 4 Lesson 23  
Module 4 Lesson 24 |
| | Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.  
  a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. | Students must use the factors to reveal and explain properties of the function, interpreting them in context. Factoring just to factor does not fully address this standard.  
Examples: (Refer to the examples in Unit 2) | |
F.IF.C.9

C. Analyze functions using different representation

Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

Example:

Describe the transformation of the quadratic parent function, \( f(x) = x^2 \), that results in the quadratic function \( g(x) = 2x^2 + 4x + 1 \).

First, rewrite \( g(x) \) into the completed-square form.

\[
g(x) = 2x^2 + 4x + 1 \\
= (2x^2 + 4x) + 1 \\
= 2(x^2 + 2x) + 1 \\
= 2(x^2 + 2x + 1) + 1 - 2 \\
= 2(x + 1)^2 - 1 \\
g(x) = 2(x + 1)^2 - 1
\]

This means that the graph of \( f(x) \) is translated 1 unit to the left, vertically stretched by a factor of 2, and translated 1 unit down.

Explanation:

This standard is taught in Algebra I and Algebra II. In Algebra I, tasks are limited to linear functions, quadratic functions, square root functions, cube root functions, piecewise-defined functions (including step functions and absolute value functions), and exponential functions with domains in the integers.

Examples (refer to examples from Topic B in addition to the examples below):

- Eureka Math:
  - Module 4 Lesson 21
  - Module 4 Lesson 22
  - Module 4 Lesson 24
F.BF.B.3  \[ \text{Build new functions from existing functions} \]

Identify the effect on the graph of replacing \( f(x) \) by \( f(x) + k \), \( kf(x) \), \( f(kx) \), and \( f(x + k) \) for specific values of \( k \) (both positive and negative); find the value of \( k \) given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. \textit{Include recognizing even and odd functions from their graphs and algebraic expressions for them.}

\[ \begin{array}{|c|c|} 
\hline
x & f(x) \\
\hline
0 & 0 \\
1 & 3 \\
2 & 8 \\
3 & 15 \\
4 & 24 \\
5 & 35 \\
\hline
\end{array} \]

One type of rectangle has lengths that are always two inches more than their widths. The function \( f \) describes the relationship between the width of this rectangle in \( x \) inches and its area, \( f(x) \), in square inches and is represented by the table below.

A second type of rectangle has lengths that are always one-half of their widths. The function \( g(x) = \frac{1}{2} x^2 \) describes the relationship between the width given in \( x \) inches and the area, \( g(x) \), given in square inches of such a rectangle.

\[ \begin{align*}
g(x) &= \frac{1}{2} x^2 \\
g(20) &= \frac{1}{2} (20)^2 \\
g(20) &= 200
\end{align*} \]

The area of the rectangle is 200 in\(^2\).

Explanation:
This standard is taught in Algebra I and Algebra II. In Algebra I, focus on vertical and horizontal translations of linear and quadratic functions. Experimenting with cases and illustrating an explanation of the effects on the graph using technology is limited to linear functions, quadratic functions, square root functions, cube root functions, piecewise functions (including step functions and absolute-value functions), and exponential functions with domains in the integers. Tasks in Algebra I do not involve recognizing even and odd functions.

Examples (refer to examples in Unit 2 in addition to the example below):

Eureka Math:
Module 4 Lesson 18
Module 4 Lesson 19
Module 4 Lesson 20
MP.1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. In Unit 3, students make sense of problems by analyzing the critical components.

Eureka Math: Module 4 Lesson 23
Module 4 Lesson 24
| **MP.2** | **Reason abstractly and quantitatively.** | Mathematically proficient students make sense of quantities and their relationships in problem situations. This module alternates between algebraic manipulation of expressions and equations and interpretation of the quantities in the relationship in terms of the context. Students must be able to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own without necessarily attending to their referents, and then to contextualize—to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning requires the habit of creating a coherent representation of the problem at hand, considering the units involved, attending to the meaning of quantities (not just how to compute them), knowing different properties of operations, and flexibility in using them. | **Eureka Math:**
Module 4 Lesson 23
Module 4 Lesson 24 |
| **MP.3** | **Construct viable arguments and critique the reasoning of others.** | Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. | **Eureka Math:**
Module 4 Lesson 19
Module 4 Lesson 20 |
| **MP.4** | **Model with mathematics.** | Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In this module, students create a function from a contextual situation described verbally, create a graph of their function, interpret key features of both the function and the graph (in the terms of the context), and answer questions related to the function and its graph. They also create a function from a data set based on a contextual situation. In Topic C, students use the full modeling cycle. They model quadratic functions presented mathematically or in a context. They explain the reasoning used in their writing or using appropriate tools, such as graphing paper, graphing calculator, or computer software. | **Eureka Math:**
Module 4 Lesson 23
Module 4 Lesson 24 |
| MP.6 | Attend to precision. | Mathematically proficient students try to communicate precisely to others. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure and labeling axes to clarify the correspondence with quantities in a problem. When calculating and reporting quantities in all topics of Unit 3, students must be precise in choosing appropriate units and use the appropriate level of precision based on the information as it is presented. When graphing, they must select an appropriate scale. | Eureka Math:  
Module 4 Lesson 22  
Module 4 Lesson 23  
Module 4 Lesson 24 |
| MP.7 | Look for and make use of structure. | Mathematically proficient students look closely to discern a pattern or structure. They can see algebraic expressions as single objects, or as a composition of several objects. In this Unit, students use the structure of expressions to find ways to rewrite them in different but equivalent forms. For example, in the expression \( x^2 + 9x + 14 \), students must see the 14 as \( 2 \times 7 \) and the 9 as \( 2 + 7 \) to find the factors of the quadratic. In relating an equation to a graph, they can see \( y = -3(x - 1)^2 + 5 \) as 5 added to a negative number times a square and realize that its value cannot be more than 5 for any real domain value. | Eureka Math:  
Module 4 Lesson 19  
Module 4 Lesson 20  
Module 4 Lesson 21 |
| MP.8 | Look for and express regularity in repeated reasoning. | Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results. | Eureka Math:  
Module 4 Lesson 19  
Module 4 Lesson 20 |
In this unit, students reconnect with and deepen their understanding of statistics and probability concepts first introduced in Grades 6, 7, and 8. There is variability in data, and this variability often makes learning from data challenging. Students develop a set of tools for understanding and interpreting variability in data, and begin to make more informed decisions from data. Students work with data distributions of various shapes, centers, and spreads. Measures of center and measures of spread are developed as ways of describing distributions. The choice of appropriate measures of center and spread is tied to distribution shape. Symmetric data distributions are summarized by the mean and mean absolute deviation or standard deviation. The median and the interquartile range summarize data distributions that are skewed. Students calculate and interpret measures of center and spread and compare data distributions using numerical measures and visual representations.

Students build on their experience with bivariate quantitative data from Grade 8; they expand their understanding of linear relationships by connecting the data distribution to a model and informally assessing the selected model using residuals and residual plots. Students explore positive and negative linear relationships and use the correlation coefficient to describe the strength and direction of linear relationships. Students also analyze bivariate categorical data using two-way frequency tables and relative frequency tables. The possible association between two categorical variables is explored by using data summarized in a table to analyze differences in conditional relative frequencies.

This module sets the stage for more extensive work with sampling and inference in later grades.

In Topic A, students observe and describe data distributions. They reconnect with their earlier study of distributions in Grade 6 by calculating measures of center and describing overall patterns or shapes. Students deepen their understanding of data distributions recognizing that the value of the mean and median are different for skewed distributions and similar for symmetrical distributions. Students select a measure of center based on the distribution shape to appropriately describe a typical value for the data distribution. Topic A moves from the general descriptions used in Grade 6 to more specific descriptions of the shape and the center of a data distribution.

<table>
<thead>
<tr>
<th>Big Idea:</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Statistics is about data.</td>
</tr>
<tr>
<td>• Graphs provide a representation of the data distribution and are used to understand the data to answer questions about the distribution.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Essential Questions:</th>
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<tbody>
<tr>
<td>• How do various representations of data lead to different interpretations of the data?</td>
</tr>
<tr>
<td>• How are center and spread of data sets described and compared?</td>
</tr>
<tr>
<td>• How is the choice made between mean and median to describe the typical value related to the shape of the data distribution?</td>
</tr>
<tr>
<td>• How do you interpret the mean?</td>
</tr>
<tr>
<td>• How are mean and median used to describe distributions of data?</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Vocabulary</th>
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<tbody>
<tr>
<td>Dot plot, histogram, box plot, mean, skewed data distribution</td>
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<tr>
<td>Galileo: Algebra 2 Module 1 Foundational Skills Assessment; Live Binders/Galileo: Topic A Assessment</td>
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<tr>
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<tbody>
<tr>
<td>8.SP.A.1</td>
<td>A. Investigate patterns of association in bivariate</td>
<td>Explanation:</td>
<td>Eureka Math: Module 6 Lessons 6-12</td>
</tr>
</tbody>
</table>
Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.

8.MP.2. Reason abstractly and quantitatively.  
8.MP.5. Use appropriate tools strategically.  
8.MP.6. Attend to precision.  
8.MP.7. Look for and make use of structure.

This standard could be incorporated in Unit 2.

Bivariate data refers to two-variable data, one to be graphed on the x-axis and the other on the y-axis. Students represent numerical data on a scatter plot, to examine relationships between variables. They analyze scatter plots to determine if the relationship is linear (positive, negative association or no association) or non-linear. Students can use tools such as those at the National Center for Educational Statistics to create a graph or generate data sets.

(http://nces.ed.gov/nceskids/createagraph/default.aspx)

Data can be expressed in years. In these situations it is helpful for the years to be “converted” to 0, 1, 2, etc. For example, the years of 1960, 1970, and 1980 could be represented as 0 (for 1960), 10 (for 1970) and 20 (for 1980).

Students recognize that points may be away from the other points (outliers) and have an effect on the linear model. **Note:** Use of the formula to identify outliers is not expected at this level.

Students recognize that not all data will have a linear association. Some associations will be non-linear as in the example below:
Examples:

• Data for 10 students’ Math and Science scores are provided in the chart. Create a scatter plot to represent the data and then describe the association between the Math and Science scores.

<table>
<thead>
<tr>
<th>Student</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Math</td>
<td>64</td>
<td>50</td>
<td>85</td>
<td>34</td>
<td>56</td>
<td>24</td>
<td>72</td>
<td>63</td>
<td>42</td>
<td>93</td>
</tr>
<tr>
<td>Science</td>
<td>68</td>
<td>70</td>
<td>83</td>
<td>33</td>
<td>60</td>
<td>27</td>
<td>74</td>
<td>63</td>
<td>40</td>
<td>96</td>
</tr>
</tbody>
</table>

Solution: This data has a positive association.

• Data for 10 students’ Math scores and the distance they live from school are provided in the table below. Create a scatter plot to represent the data and then describe the association between the Math scores and the distance they live from school.

<table>
<thead>
<tr>
<th>Student</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Math</td>
<td>64</td>
<td>50</td>
<td>85</td>
<td>34</td>
<td>56</td>
<td>24</td>
<td>72</td>
<td>63</td>
<td>42</td>
<td>93</td>
</tr>
<tr>
<td>Distance from School (miles)</td>
<td>0.5</td>
<td>18</td>
<td>1</td>
<td>2.3</td>
<td>3.4</td>
<td>0.2</td>
<td>2.5</td>
<td>1.6</td>
<td>0.8</td>
<td>2.5</td>
</tr>
</tbody>
</table>

Solution: There is no association between the math score and the distance a student lives from school.

• Data from a local fast food restaurant is provided showing the number of staff members and the average time for filling an order are provided in the table below. Create a scatter plot to represent the data and then describe the association between the number of staff and the average time for filling an order.

<table>
<thead>
<tr>
<th>Number of Staff</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average time to fill order (seconds)</td>
<td>56</td>
<td>24</td>
<td>72</td>
<td>63</td>
<td>42</td>
<td>93</td>
</tr>
</tbody>
</table>

Solution: There is a positive association.

• The chart below lists the life expectancy in years for people in the United States every five years from 1970 to 2005. What
would you expect the life expectancy of a person in the United States to be in 2010, 2015, and 2020 based upon this data? Explain how you determined your values.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Life Expectancy (in years)</td>
<td>70.8</td>
<td>72.6</td>
<td>73.7</td>
<td>74.7</td>
<td>75.4</td>
<td>75.8</td>
<td>76.8</td>
<td>77.4</td>
</tr>
</tbody>
</table>

Solution: Answers will vary.

- The scatter plot below was constructed using data on length in inches \( (x) \) and weight in pounds \( (y) \) for a sample of alligators. Write a few sentences describing the relationship between weight and length for these alligators. Are there any noticeable clusters or outliers?

Solution: Answers will vary. Possible response: There appears to be a positive relationship between length and weight, but the relationship is not linear. Weight tends to increase as length increases. There are three observations that stand out as outliers. These correspond to alligators that are much bigger in terms of both length and weight than the other alligators in the sample. Without these three alligators, the relationship between length and weight would look linear. It might be
possible to use a line to model the relationship between weight and length for alligators that have lengths of less than 100 inches.

<table>
<thead>
<tr>
<th>8.SP.A.2</th>
<th>A. Investigate patterns of association in bivariate data</th>
<th>Explanation:</th>
<th>Eureka Math: Module 6 Lessons 6-12</th>
<th>Big Ideas: Sections: 9.2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.</td>
<td>This standard could be incorporated in Unit 2. Students understand that a straight line can represent a scatter plot with linear association. The most appropriate linear model is the line that comes closest to most data points. The use of linear regression is not expected. If there is a linear relationship, students draw a linear model. Given a linear model, students write an equation.</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>• The capacity of the fuel tank in a car is 13.5 gallons. The table below shows the number of miles traveled and how many gallons of gas are left in the tank. Describe the relationship between the variables. If the data is linear, determine a line of best fit. Do you think the line represents a good fit for the data set? Why or why not? What is the average fuel efficiency of the car in miles per gallon?</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>![Miles Traveled vs. Gallons Used Table]</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Miles Traveled</td>
<td>0</td>
<td>75</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Gallons Used</td>
<td>0</td>
<td>2.3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>8.SP.A.3</th>
<th>A. Investigate patterns of association in bivariate data</th>
<th>Explanation:</th>
<th>Eureka Math: Module 6 Lessons 6-12</th>
<th>Big Ideas: Sections: 9.2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.</td>
<td>This standard could be incorporated in Unit 2. Linear models can be represented with a linear equation. Students interpret the slope and y-intercept of the line in the context of the problem.</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>8.MP.2. Reason abstractly and quantitatively.</td>
<td>Examples:</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Given the following data from students’ math scores and</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
$8.MP.5$. Use appropriate tools strategically.
$8.MP.6$. Attend to precision.
$8.MP.7$. Look for and make use of structure.

<table>
<thead>
<tr>
<th>Absences</th>
<th>Math Scores</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>65</td>
</tr>
<tr>
<td>5</td>
<td>50</td>
</tr>
<tr>
<td>1</td>
<td>95</td>
</tr>
<tr>
<td>1</td>
<td>85</td>
</tr>
<tr>
<td>3</td>
<td>80</td>
</tr>
<tr>
<td>6</td>
<td>34</td>
</tr>
<tr>
<td>5</td>
<td>70</td>
</tr>
<tr>
<td>3</td>
<td>56</td>
</tr>
<tr>
<td>0</td>
<td>100</td>
</tr>
<tr>
<td>7</td>
<td>74</td>
</tr>
<tr>
<td>0</td>
<td>45</td>
</tr>
<tr>
<td>2</td>
<td>71</td>
</tr>
<tr>
<td>9</td>
<td>30</td>
</tr>
<tr>
<td>0</td>
<td>95</td>
</tr>
<tr>
<td>6</td>
<td>55</td>
</tr>
<tr>
<td>0</td>
<td>42</td>
</tr>
<tr>
<td>2</td>
<td>90</td>
</tr>
<tr>
<td>0</td>
<td>92</td>
</tr>
<tr>
<td>5</td>
<td>60</td>
</tr>
<tr>
<td>7</td>
<td>50</td>
</tr>
<tr>
<td>9</td>
<td>10</td>
</tr>
<tr>
<td>1</td>
<td>80</td>
</tr>
</tbody>
</table>

- Create a scatter plot that represents the data.
- Draw a linear model paying attention to the closeness of the data points on either side of the line.
- From the linear model, determine an approximate linear equation that models the given data.
- What does the slope and y-intercept represent in the context of the problem?
- Use your prediction equation to predict the math score of a student with 4 absences. How does this prediction compare to the line of best fit?

**Solution:**
c) 

\( \text{(about } y = -\frac{25}{3}x + 95) \)

d) Students should recognize that 95 represents the \( y \)-intercept and \(-25/3\) represents the slope of the line. In the context of the problem, the \( y \)-intercept represents the math score a student with 0 absences could expect. The slope indicates that the math scores decreased 25 points for every 3 absences.

e) A student with 4 absences should expect to receive a math score of about 62. This point would fall on the line.

- The following scatter plot shows the burn time for candles of various weights.
  - Draw a line of best fit on the scatter plot.
  - Write an equation for the line of best fit you drew.
  - Explain the meaning of the slope and \( y \)-intercept in the context of the problem.
  - Use your prediction equation to predict the burn time for a candle that weighs 40 ounces.
  - If the candle burns out at 500 hours, predict how much the candle weighs.
  - What do you think would happen if we changed the graph above so that the burn time was on the \( x \)-axis
and weight was on the y-axis? Would our data still resemble a line? What would happen to the slope and y-intercept of the line of best fit?

Solution: Answer will vary.

8.SP.A.4

A. Investigate patterns of association in bivariate data

Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?

8.MP.2. Reason abstractly and quantitatively.
8.MP.3. Construct viable arguments and critique the

Explanation:
Students understand that a two-way table provides a way to organize data between two categorical variables. Data for both categories needs to be collected from each subject. Students calculate the relative frequencies to describe associations.

Examples:
- Twenty-five students were surveyed and asked if they received an allowance and if they did chores. The table below summarizes their responses.

<table>
<thead>
<tr>
<th>Receive Allowance</th>
<th>No Allowance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Do Chores</td>
<td>15</td>
</tr>
<tr>
<td>Do Not Do Chores</td>
<td>3</td>
</tr>
</tbody>
</table>

Gr 8 Eureka Math: Module 6 Lessons 13-14
Gr 8 Big Ideas: Sections: 9.3
Of the students who do chores, what percent do not receive an allowance?

Solution: 5 of the 20 students who do chores do not receive an allowance, which is 25%

- The table illustrates the results when 100 students were asked the survey questions: “Do you have a curfew?” and “Do you have assigned chores?” Is there evidence that those who have a curfew also tend to have chores?

<table>
<thead>
<tr>
<th>Chores</th>
<th>Curfew</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Yes</td>
</tr>
<tr>
<td>Yes</td>
<td>40</td>
</tr>
<tr>
<td>No</td>
<td>10</td>
</tr>
</tbody>
</table>

Solution: Of the students who answered that they had a curfew, 40 had chores and 10 did not. Of the students who answered they did not have a curfew, 10 had chores and 40 did not. From this sample, there appears to be a positive correlation between having a curfew and having chores.

- Suppose a sample of 400 participants (teachers and student) was randomly selected from the middle schools and high schools in a large city. These participants responded to the question:
  - Which type of movie do you prefer to watch?
    - Action
    - Drama
    - Science Fiction
    - Comedy

Movie preference and status (teacher/student) was recorded for each participant. The results of the survey are summarized in the table below.
• Two variables were recorded. Are these variables categorical or numerical?
  
  Solution: Both variables are categorical.

• Create a row relative frequency table for the data.
  
  Solution:

<table>
<thead>
<tr>
<th>Movie Preference</th>
<th>Action</th>
<th>Drama</th>
<th>Science Fiction</th>
<th>Comedy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Student</td>
<td>120</td>
<td>60</td>
<td>30</td>
<td>90</td>
</tr>
<tr>
<td>Teacher</td>
<td>40</td>
<td>20</td>
<td>10</td>
<td>30</td>
</tr>
<tr>
<td>Total</td>
<td>160</td>
<td>80</td>
<td>40</td>
<td>120</td>
</tr>
</tbody>
</table>

• Is a participant’s status related to what type of movie he or she would prefer to watch? Why or why not?
  
  Solution: No, because teachers are just as likely to prefer each movie type as students are, based on the row relative frequencies.

• Is there an association between the two variables?
  
  Solution: There is no association between the two variables because the row relative frequencies for each movie type are the same for both the teacher and student rows. No association means that knowing the value of one variable does not tell you anything about the value of the other variable.

• Create a column relative frequency table for the data.

<table>
<thead>
<tr>
<th>Movie Preference</th>
<th>Action</th>
<th>Drama</th>
<th>Science Fiction</th>
<th>Comedy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Student</td>
<td>0.40</td>
<td>0.20</td>
<td>0.10</td>
<td>0.30</td>
</tr>
<tr>
<td>Teacher</td>
<td>a) 0.40 b) 0.20 c) 0.10 d) 0.30</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>0.75</td>
<td>0.75</td>
<td>0.75</td>
<td>0.75</td>
</tr>
<tr>
<td>S.ID.A.1</td>
<td>A. Summarize, represent, and interpret data on a single count or measurement variable</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>----------</td>
<td>-----------------------------------------------------------------------------------</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Represent data with plots on the real number line (dot plots, histograms, and box plots).</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td><em>This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.</em></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Explanation:</th>
</tr>
</thead>
<tbody>
<tr>
<td>A statistical process is a problem-solving process consisting of four steps:</td>
</tr>
<tr>
<td>1. Formulating a statistical question that anticipates variability and can be answered by data.</td>
</tr>
<tr>
<td>2. Designing and implementing a plan that collects appropriate data.</td>
</tr>
<tr>
<td>3. Analyzing the data by graphical and/or numerical methods.</td>
</tr>
<tr>
<td>4. Interpreting the analysis in the context of the original question.</td>
</tr>
<tr>
<td>The four-step statistical process was introduced in Grade 6, with the recognition of statistical questions. In middle school, students describe center and spread in a data distribution. In Algebra I, students need to become proficient in the first step of generating meaningful questions and choose a summary statistic appropriate to the characteristic of the data distribution, such as, the shape of the distribution or the existence of extreme data points.</td>
</tr>
<tr>
<td>Students will graph numerical data on a real number line using dot plots, histograms, and box plots.</td>
</tr>
<tr>
<td>• Analyze the strengths and weakness inherent in each type of plot by comparing different plots of the same data.</td>
</tr>
<tr>
<td>• Describe and give a simple interpretation of a graphical representation of data.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Example:</th>
</tr>
</thead>
</table>
The following data set shows the number of songs downloaded in one week by each student in Mrs. Jones class: 10, 20, 12, 14, 12, 27, 88, 2, 7, 20, 16, 16, 32, 25, 15, 4, 0, 15, 6. Choose and create a data display to represent the data. Explain why you chose the data display.

**Students should use appropriate tools strategically.**
- The use of a graphing calculator to represent the data should be utilized.
- Emphasis must be on analyzing the data display to make decisions, not on making the display itself.

**Examples:**

Dot plots: A plot of each data value on a scale or number line.
- Transportation officials collect data on flight delays (the number of minutes past the scheduled departure time that a flight takes off). Consider the dot plot of the delay times for sixty BigAir flights during December 2012.

[Dot plot image]

- What do you think this graph is telling us about the flight delays for these sixty flights?
  - *Most flights are delayed for 15 minutes; some are delayed for a longer time.*

- Can you think of a reason why the data presented by this graph provides important information? Who might be interested in this data distribution?
  - *If flights are late, travelers would not select this airline. BigAir and travelers using this airline would be interested in this information.*

- Based on your previous work with dot plots, would you describe this dot plot as representing a
Histograms: A graph of data that groups the data based on intervals and represents the data in each interval by a bar.

- The following histogram represents the age distribution of the population of Kenya in 2010.

---

What do you think this graph is telling us about the population of Kenya?
- A large percentage of the people in Kenya are ages 10 or younger.

Why might we want to study the data represented by this graph?
- It tells us about Kenya and its challenges based on its population and demographics. Caring for and educating young people are major challenges for this country.

Based on your previous work with histograms, would you describe this histogram as representing a symmetrical or a skewed distribution?
- Skewed; it has a tail to the right
Box plots: A graph that provides a picture of the data ordered and divided into four intervals that each contains approximately 25% of the data.

- Thirty students from River City High School were asked how many pets they owned. The following box-plot was prepared from their answers.

```
<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2</td>
<td>4</td>
<td>6</td>
<td>8</td>
</tr>
</tbody>
</table>
```

Number of Pets

- What does the box-plot tell us about the number of pets owned by the thirty students at River City HS?
  - 50% of students own between 1 and 5 pets.

- Why might understanding the data behind this graph be important?
  - Understanding the data is important for planning special events involving pets and understanding interests of a group of people.

<table>
<thead>
<tr>
<th>S.ID.A.2</th>
<th>A. Summarize, represent, and interpret data on a single count or measurement variable</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two</td>
</tr>
<tr>
<td></td>
<td>Given two sets of data or two graphs, students</td>
</tr>
<tr>
<td></td>
<td>- Identify the similarities and differences in shape, center and</td>
</tr>
</tbody>
</table>

**Explanation:**

Students may use spreadsheets, graphing calculators and statistical software for calculations, summaries, and comparisons of data sets.

**Eureka Math:**

Module 2 Lesson 2,3

**Big Ideas:**

Pg. 596
Pg. 598 #17, 21
This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.

- **Spread**
  - Compare data sets and summarize the similarities and difference between the shape, and measures of center and spreads of the data sets
  - Use the correct measure of center and spread to describe a distribution that is symmetric or skewed
  - Identify outliers and their effects on data sets

The mean and standard deviation are most commonly used to describe sets of data. However, if the distribution is extremely skewed and/or has outliers, it is best to use the median and the interquartile range to describe the distribution since these measures are not sensitive to outliers.

**Examples:**
- You are planning to take on a part time job as a waiter at a local restaurant. During your interview, the boss told you that their best waitress, Jenni, made an average of $70 a night in tips last week. However, when you asked Jenni about this, she said that she made an average of only $50 per night last week. She provides you with a copy of her nightly tip amounts from last week. Calculate the mean and the median tip amount.

<table>
<thead>
<tr>
<th>Day</th>
<th>Tip Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sunday</td>
<td>$50</td>
</tr>
<tr>
<td>Monday</td>
<td>$45</td>
</tr>
<tr>
<td>Wednesday</td>
<td>$48</td>
</tr>
<tr>
<td>Friday</td>
<td>$125</td>
</tr>
<tr>
<td>Saturday</td>
<td>$85</td>
</tr>
</tbody>
</table>

- Which value is Jenni’s boss using to describe the average tip? Why do you think he chose this value?
- Which value is Jenni using? Why do you think she chose this value?
- Which value best describes the typical amount of tips per night? Explain why.

This standard is revisited in Topic B.

pp. 599-606

IXL Interpret box plots
• Delia wanted to find the best type of fertilizer for her tomato plants. She purchased three types of fertilizer and used each on a set of seedlings. After 10 days, she measured the heights (in cm) of each set of seedlings. The data she collected is shown below.

<table>
<thead>
<tr>
<th>Fertilizer A</th>
<th>Fertilizer B</th>
<th>Fertilizer C</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.1</td>
<td>13.0</td>
<td>10.5</td>
</tr>
<tr>
<td>6.5</td>
<td>9.2</td>
<td>11.8</td>
</tr>
<tr>
<td>1.0</td>
<td>5.6</td>
<td>15.5</td>
</tr>
<tr>
<td>5.0</td>
<td>8.4</td>
<td>14.7</td>
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<td>4.5</td>
<td>7.2</td>
<td>11.9</td>
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<td>5.2</td>
<td>12.1</td>
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<tr>
<td>3.2</td>
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<tr>
<td>4.6</td>
<td>14.0</td>
<td>12.7</td>
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<tr>
<td>2.4</td>
<td>15.3</td>
<td>9.9</td>
</tr>
<tr>
<td>5.5</td>
<td>6.3</td>
<td>10.3</td>
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<tr>
<td>3.0</td>
<td>8.7</td>
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<td>1.5</td>
<td>11.3</td>
<td>15.8</td>
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<tr>
<td>6.2</td>
<td>17.0</td>
<td>9.5</td>
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<tr>
<td>6.9</td>
<td>13.5</td>
<td>13.2</td>
</tr>
<tr>
<td>5.6</td>
<td>14.2</td>
<td>9.7</td>
</tr>
</tbody>
</table>

- Construct box plots to analyze the data. Write a brief description comparing the three types of fertilizer. Which fertilizer do you recommend that Delia use? Explain your answer.

- The two data sets below depict the housing prices sold in the King River area and Toby Ranch areas of Pinal County, Arizona. Based on the prices below which price range can be expected for a home purchased in Toby Ranch? In the King River area? In Pinal County?

- King River area {1.2 million, 242000, 265500, 140000, 281000, 265000, 211000}
- Toby Ranch homes {5 million, 154000, 250000, 250000, 200000, 160000, 190000}

- Given a set of test scores {99, 96, 94, 93, 90, 88, 86, 77, 70, 68}, find the mean, median and standard deviation. Explain how the values vary about the mean and median. What information does this give the teacher?

<table>
<thead>
<tr>
<th>S.ID.A.3</th>
<th>A. Summarize, represent, and interpret data on a single count or measurement variable</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects</td>
</tr>
</tbody>
</table>

**Explanation:**
Students may use spreadsheets, graphing calculators and statistical software for calculations, summaries, and comparisons of data sets.

**Students understand and use the context of the data to explain why its distribution takes on a particular shape (e.g. Why is the data**

**Eureka Math:**
Module 2 Lesson 1-3

**Big Ideas:**
pp. 585 - 592
pp. 599 – 606
of extreme data points (outliers).

*This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.*

Examples:
- Why does the shape of the distribution of incomes for professional athletes tend to be skewed to the right?
- Why does the shape of the distribution of test scores on a really easy test tend to be skewed to the left?
- Why does the shape of the distribution of heights of the students at your school tend to be symmetrical?

Students understand that the higher the value of a measure of variability, the more spread out the data set is. Measures of variability are range (100% of data), standard deviation (68-95-99.7% of data), and interquartile range (50% of data).

Example:
- On last week’s math test, Mrs. Smith’s class had an average of 83 points with a standard deviation of 8 points. Mr. Tucker’s class had an average of 78 points with a standard deviation of 4 points. Which class was more consistent with their test scores? How do you know?

Students explain the effect of any outliers on the shape, center, and spread of the data sets.

Example:
- The heights of Washington High School’s basketball players are: 5ft. 4in, 5ft. 7in, 5ft. 5in, 5ft. 3in, and 5ft. 7in. A student transfers to Washington High and joins the basketball team. Her height is 6ft. 10in.
  a. What is the mean height of the team before the new player transfers in? What is the median height?
  b. What is the mean height after the new player transfers? What is the median height?
  c. What affect does her height have on the team’s height distribution and stats (center and spread)?
  d. How many players are taller than the new mean team height?
| **MP.1** | **Make sense of problems and persevere in solving them.** | Students choose an appropriate method of analysis based on problem context. They consider how the data were collected and how data can be summarized to answer statistical questions. Students select a graphical display appropriate to the problem context. They select numerical summaries appropriate to the shape of the data distribution. Students use multiple representations and numerical summaries and then determine the most appropriate representation and summary for a given data distribution. **Example:** Students are provided three data sets and asked to construct dot plots for each and use them to study the distributions. A symmetrical or a nearly symmetrical distribution emerges from the first data set. Students determine that the mean and median are nearly the same in this distribution. In the second and third data sets, a non-symmetrical distribution is given. Students determine that the mean and the median are not the same. They are asked to explain why the two measures of center are not equal. | **Eureka Math:** Module 2 Lesson 2 |
| **MP.2** | **Reason abstractly and quantitatively.** | Students pose statistical questions and reason about how to collect and interpret data in order to answer these questions. Students form summaries of data using graphs, two-way tables, and other representations that are appropriate for a given context and the statistical question they are trying to answer. Students reason about whether two variables are associated by considering conditional relative frequencies. **Examples:** • How does the dot plot for the juniors differ from the dot plot for the seniors? What might explain the difference between the dot plots for juniors and seniors? • How would you describe the typical number of miles walked by a junior? | **Eureka Math:** Module 2 Lesson 1 Module 2 Lesson 3 |
| **MP.4** | **Model with mathematics.** | Students construct and interpret two-way tables to summarize bivariate categorical data. Students graph bivariate numerical data using a scatterplot and propose a linear, exponential, quadratic, or other model to describe the relationship between two numerical | **Eureka Math:** Module 2 Lesson 3 |
variables. Students use residuals and residual plots to assess if a linear model is an appropriate way to summarize the relationship between two numerical variables.

**Example:**
The concept of balance is developed by using a dot plot and a representation of the data as equal weights along a number line. Balance would be a position on the number line in which the sum of the distances on the right and the sum of the distances on the left are equal. Students model with mathematics as they verify this position.
**8th Grade Algebra I Semester 2 (Quarter 4)**

**Unit 4: Descriptive Statistics (25 days)**

**Topic B: Describing Variability and Comparing Distributions**

In Topic B, students reconnect with methods for describing variability first seen in Grade 6. Topic B deepens students' understanding of measures of variability by connecting a measure of the center of a data distribution to an appropriate measure of variability. The mean is used as a measure of center when the distribution is more symmetrical. Students calculate and interpret the mean absolute deviation and the standard deviation to describe variability for data distributions that are approximately symmetric. The median is used as a measure of center for distributions that are more skewed and students interpret the interquartile range as a measure of variability for data distributions that are not symmetric. Students match histograms to box plots for various distributions based on an understanding of center and variability. Students describe data distributions in terms of shape, a measure of center, and a measure of variability from the center.

### Big Idea:
- The larger the standard deviation, the greater the variability of the data set.
- Data consists of structure plus variability.

### Essential Questions:
- What measure of center and measure of variability is a better description for skewed distributions?
- Is data distribution represented by the box plot a skewed distribution?
- What information is displayed in a box plot?
- What does standard deviation measure?
- How does the spread of the distribution relate to the value of the standard deviation?

### Vocabulary
- Deviation from the mean, standard deviation, outliers, interquartile range (IQR), skewed

### Assessments
- Live Binders/Galileo: Topic B Assessment

<table>
<thead>
<tr>
<th>Standard</th>
<th>AZ College and Career Readiness Standards</th>
<th>Explanations &amp; Examples</th>
<th>Resources</th>
</tr>
</thead>
</table>
| S.ID.A.1 | **A. Summarize, represent, and interpret data on a single count or measurement variable**
Represent data with plots on the real number line (dot plots, histograms, and box plots).

*This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.*

**Explanation:**
A statistical process is a problem-solving process consisting of four steps:
1. Formulating a statistical question that anticipates variability and can be answered by data.
2. Designing and implementing a plan that collects appropriate data.
3. Analyzing the data by graphical and/or numerical methods.
4. Interpreting the analysis in the context of the original question.

| **Eureka Math:** Module 2 Lesson 4-7 |
| **Big Ideas:**
Pg 593 exploration #1
Pg 594 ex 1
Pg 601 ex 2 |
| **Other:**
Illuminations: [Box Plotter](#) |
The four-step statistical process was introduced in Grade 6, with the recognition of statistical questions. In middle school, students describe center and spread in a data distribution. In Algebra I, students need to become proficient in the first step of generating meaningful questions and choose a summary statistic appropriate to the characteristic of the data distribution, such as, the shape of the distribution or the existence of extreme data points.

Students will graph numerical data on a real number line using dot plots, histograms, and box plots.
- Analyze the strengths and weakness inherent in each type of plot by comparing different plots of the same data.
- Describe and give a simple interpretation of a graphical representation of data.

Example (refer to examples in Topic A in addition to the example below):
- Ten members of a high school girls’ basketball team were asked how many hours they studied in a typical week. Their responses (in hours) were 20, 13, 10, 6, 13, 10, 13, 11, 11, 10. Using the axis given below, draw a dot plot of these values.

![Dot plot example](image)

Students should use appropriate tools strategically.
- The use of a graphing calculator to represent the data should be utilized.
- Emphasis must be on analyzing the data display to make decisions, not on making the display itself.

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<table>
<thead>
<tr>
<th>S.ID.A.2</th>
<th>A. Summarize, represent, and interpret data on a single count or measurement variable</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Explanation: Students may use spreadsheets, graphing calculators and statistical software for calculations, summaries, and comparisons of data sets.</td>
</tr>
</tbody>
</table>

Illuminations: [Histogram Tool](https://illuminations.nctm.org/Activity.aspx?id=5075)

Eureka Math: Module 2 Lesson 5-8
Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.

<table>
<thead>
<tr>
<th>Given two sets of data or two graphs, students</th>
<th>The mean and standard deviation are most commonly used to describe sets of data. However, if the distribution is extremely skewed and/or has outliers, it is best to use the median and the interquartile range to describe the distribution since these measures are not sensitive to outliers.</th>
</tr>
</thead>
</table>
| • Identify the similarities and differences in shape, center and spread | Comparing histograms:  
  • How do the shapes of the two histograms differ? |
| • Compare data sets and summarize the similarities and difference between the shape, and measures of center and spreads of the data sets | Comparing Box Plots:  
  • What information is displayed in a box plot?  
  • Can we find this same information in histograms? |
| • Use the correct measure of center and spread to describe a distribution that is symmetric or skewed | Examples (refer to examples in Topic A in addition to the example below):  
  • At a track meet there were three men’s 100m races. The sprinter’s times were recorded to the 1/10 of a second. The results of the three races are shown in the dot plots below. |

**Big Ideas:**
- Pg 599 exploration 2
- Pg 602 ex 3
- Pg 604 ex 4
- Pg 605 #15-16
- Pg 606 #19-23

**Other:**
- Illuminations – [Mean and Median](#)
Remember that the size of the standard deviation is related to the sizes of the deviations from the mean. Without doing any calculations, indicate which of the three races has the smallest standard deviation of times. Justify your answer.

Race 3 as several race times are clustered around the mean.

Which race had the largest standard deviation of times? (Again, don’t do any calculations!) Justify your answer.

Race 2 as the race times are spread out from the mean.

Roughly what would be the standard deviation in Race 1? (Remember that the standard deviation is a typical deviation from the mean. So, here you are looking for a typical deviation from the mean, in seconds, for Race 1.)

Around 0.5-1.0 second would be a sensible answer.

Use your calculator to find the mean and the standard deviation for each of the three races. Write your answers in the table below to the nearest
A. Summarize, represent, and interpret data on a single count or measurement variable

Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).

This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.

Explanation:

Students may use spreadsheets, graphing calculators and statistical software for calculations, summaries, and comparisons of data sets.

Students understand and use the context of the data to explain why its distribution takes on a particular shape (e.g. Why is the data skewed? Are there outliers?)

Examples:

• Why does the shape of the distribution of incomes for professional athletes tend to be skewed to the right?
• Why does the shape of the distribution of test scores on a really easy test tend to be skewed to the left?
• Why does the shape of the distribution of heights of the students at your school tend to be symmetrical?

Students understand that the higher the value of a measure of variability, the more spread out the data set is. Measures of variability are range (100% of data), standard deviation (68-95-99.7% of data), and interquartile range (50% of data).

Example (refer to the examples in Topic A in addition to the example below):

• Jenna has bought a new hybrid car. Each week for a period of seven weeks, she has noted the fuel efficiency (in miles per gallon) of her car. The results are shown below. 45 44 43 44 45 44 43

<table>
<thead>
<tr>
<th>Race</th>
<th>Mean</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Race 1</td>
<td>11.725</td>
<td>0.707</td>
</tr>
<tr>
<td>Race 2</td>
<td>11.813</td>
<td>1.013</td>
</tr>
<tr>
<td>Race 3</td>
<td>11.737</td>
<td>0.741</td>
</tr>
</tbody>
</table>

Eureka Math:
Module 2 Lesson 4-7
Big Ideas:
Pp 585-592
Pg 595 ex 2
Pg 596 ex 3
Practice pp 597-598
Pg 600 ex 1
Calculate the standard deviation of these results to the nearest hundredth. Be sure to show your work.

- The mean is 44.
- The deviations from the mean are 1, -1, 0, 1, 0, -1.
- The squared deviations from the mean are 1, 1, 0, 1, 0, 1.
- The sum of the squared deviations is 4. \( n=7; \ \frac{46}{7}=6.67 \)
- The standard deviation is \( \sqrt{6.67}=0.82 \) miles per gallon.

What is the meaning of the standard deviation you found?
- The standard deviation, 0.82 miles per gallon, is a typical deviation of a weekly fuel efficiency value from the mean weekly fuel efficiency.

Students explain the effect of any outliers on the shape, center, and spread of the data sets.

Example:
- The heights of Washington High School’s basketball players are: 5ft. 4in, 5ft. 7in, 5ft. 5in, 5ft. 3in, and 5ft. 7in. A student transfers to Washington High and joins the basketball team. Her height is 6ft. 10in.
  a. What is the mean height of the team before the new player transfers in? What is the median height?
  b. What is the mean height after the new player transfers? What is the median height?
  c. What affect does her height have on the team’s height distribution and stats (center and spread)?
  d. How many players are taller than the new mean team height?
  e. Which measure of center most accurately describes the team’s average height? Explain.

**MP.1**

*Make sense of problems and persevere in solving them.*

Students choose an appropriate method of analysis based on problem context. They consider how the data were collected and how data can be summarized to answer statistical questions. Students select a graphical display appropriate to the problem context. They select **Eureka Math:**

Module 2 Lesson 4
Module 2 Lesson 6
<p>| | | |</p>
<table>
<thead>
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<th></th>
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</thead>
<tbody>
<tr>
<td></td>
<td>numerical summaries appropriate to the shape of the data distribution. Students use multiple representations and numerical summaries and then determine the most appropriate representation and summary for a given data distribution.</td>
<td></td>
</tr>
<tr>
<td><strong>MP.2</strong></td>
<td><strong>Reason abstractly and quantitatively.</strong></td>
<td>Students pose statistical questions and reason about how to collect and interpret data in order to answer these questions. Students form summaries of data using graphs, two-way tables, and other representations that are appropriate for a given context and the statistical question they are trying to answer. Students reason about whether two variables are associated by considering conditional relative frequencies.</td>
</tr>
</tbody>
</table>
| **MP.3** | **Construct viable arguments and critique the reasoning of others.** | Students examine the shape, center, and variability of a data distribution and use characteristics of the data distribution to communicate the answer to a statistical question in the form of a poster presentation. Students also have an opportunity to critique poster presentations made by other students. | **Eureka Math:** Module 2 Lesson 4
Module 2 Lesson 5
Module 2 Lesson 7
Module 2 Lesson 8 |
# 8th Grade Algebra I Semester 2 (Quarter 4)

## Unit 4: Descriptive Statistics (25 days)

### Topic C: Categorical Data on Two Variables

In Topic C, students reconnect with previous work in Grade 8 involving categorical data. Students use a two-way frequency table to organize data on two categorical variables. Students calculate the conditional relative frequencies from the frequency table. They explore a possible association between two categorical variables using differences in conditional relative frequencies. Students also come to understand the distinction between association between two categorical variables and a causal relationship between two variables. This provides a foundation for work on sampling and inference in later grades.

### Big Idea:

- Data in a two-way frequency table can be summarized using relative frequencies in the context of the data.
- Association does not imply causation.

### Essential Questions:

- What is the difference between categorical data and numerical data?
- How is relative frequency calculated?

### Vocabulary

- Two-way frequency table
- Marginal frequency
- Categorical data
- Numerical data
- Joint frequency
- Relative frequency table
- Conditional relative frequency

### Assessments

- Live Binders/Galileo: Topic C Assessment

### Standard

<table>
<thead>
<tr>
<th>Standard</th>
<th>AZ College and Career Readiness Standards</th>
<th>Explanations &amp; Examples</th>
<th>Resources</th>
</tr>
</thead>
</table>
| S.ID.B.5  | B. Summarize, represent, and interpret data on a single count or measurement variable | **Explanation:** Students may use spreadsheets, graphing calculators and statistical software for calculations, summaries, and comparisons of data sets. When students are proficient with analyzing two-way frequency tables, build upon their understanding to develop the vocabulary.  
- The table entries are the joint frequencies of how many subjects displayed the respective cross-classified values.  
- Row totals and column totals constitute the marginal frequencies.  
- Dividing joint or marginal frequencies by the total number of subjects define relative frequencies, respectively. | Eureka Math: Module 2 Lesson 9-11  
Big Ideas: pp. 609-616  
Texas Instruments: Two-way tables |
Conditional relative frequencies are determined by focusing on a specific row or column of the table. They are particularly useful in determining any associations between the two variables.

**Two-way Frequency Table**
A two-way frequency table is shown below displaying the relationship between age and baldness. We took a sample of 100 male subjects, and determined who is or is not bald. We also recorded the age of the male subjects by categories.

![Two-way Frequency Table](image1)

The total row and total column entries in the table above report the marginal frequencies, while entries in the body of the table are the joint frequencies.

**Two-way Relative Frequency Table**
The relative frequencies in the body of the table are called conditional relative frequencies.

![Two-way Relative Frequency Table](image2)
The data used in this lesson were obtained from the Census at School project. Census at School is further explained at the American Statistical Association website* and can be a source for data that might interest teenagers.

*www.amstat.org/censusatschool

Students are flexible in identifying and interpreting the information from a two-way frequency table. They complete calculations to determine frequencies and use those frequencies to describe and compare.

Example:
At the PHX Zoo 23 interns were asked their preference of where they would like to work. There were three choices: African Region, Aviary, or North American Region. There were 13 who preferred the African Region, 5 of them were male. There were 6 who preferred the Aviary, 2 males and 4 females. A total of 4 preferred the North American Region and only 1 of them was female.

a. Use the information on the PHX Zoo Internship to create a two-way frequency table.

b. Use the two-way frequency table from the PHX Zoo Internship, to calculate:
   i. The percentage of males who prefer the African Region
   ii. The percentage of females who prefer the African Region

c. How does the percentage of males who prefer the African Region compare to the percentage of females who prefer the African Region?

d. 15% of the paid employees are male and work in the Aviary. How does that compare to the interns who are male and prefer to work in the Aviary? Explain how you made your comparison.

S.ID.C.9

C. Interpret linear models
Distinguish between correlation and causation.

This is a modeling standard which means students

Explanation:
Differences in conditional relative frequencies are used as evidence of possible association. Instructors should challenge students to think critically about the meaning of an association between two

Eureka Math:
Module 2 Lesson 11
**choose and use appropriate mathematics to analyze situations.** Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.

Some data leads observers to believe that there is a cause and effect relationship when a strong relationship is observed. Students should be careful not to assume that correlation implies causation. The determination that one thing causes another requires a controlled randomized experiment.

**Discuss examples of questions that investigate association:**

- A study found a strong, positive correlation between the number of cars owned and the length of one’s life. Larry concludes that owning more cars means you will live longer. Does this seem reasonable? Explain your answer.
- Choose two variables that could be correlated because one is the cause of the other; defend and justify selection of variables.
  
- If dogs are classified as large, medium, or small based on weight, are small dogs more likely to pass an obedience course?
- If users of a social network are classed as active, average, or inactive, is a person classified as an active user more likely to be a good writer than those classified in the other categories?

**Then discuss the evidence that supports association:**

- There is strong evidence of association when there is a noticeable difference in conditional relative frequencies.
- What is a noticeable difference in conditional relative frequencies? This is subjective; students should use their best judgment at this time. Evaluating the differences more formally is discussed in the Grade 11 and 12.

**Big Ideas:**
- Pg. 205 Ex 5
- Pg. 207 #21-24
Examples:

• Diane did a study for a health class about the effects of a student’s end-of-year math test scores on height. Based on a graph of her data, she found that there was a direct relationship between students’ math scores and height. She concluded that “doing well on your end-of-course math tests makes you tall.” Is this conclusion justified? Explain any flaws in Diane’s reasoning.

• Students were given the opportunity to prepare for a college placement test in mathematics by taking a review course. Not all students took advantage of this opportunity. The following results were obtained from a random sample of students who took the placement test:

<table>
<thead>
<tr>
<th>Took Review Course</th>
<th>Placed in Math 200</th>
<th>Placed in Math 100</th>
<th>Placed in Math 50</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Took Review Course</td>
<td>40</td>
<td>13</td>
<td>7</td>
<td>60</td>
</tr>
<tr>
<td>Did not take Review Course</td>
<td>10</td>
<td>15</td>
<td>15</td>
<td>40</td>
</tr>
<tr>
<td>Total</td>
<td>50</td>
<td>28</td>
<td>22</td>
<td>100</td>
</tr>
</tbody>
</table>

This example introduces the important idea that you should not infer a cause-and-effect relationship from an association between two categorical variables.

Pose the following questions to the class. Let students discuss their ideas.
• Do you think there is an association between taking the review course and a student’s placement in a math class?
• If you knew that a student took a review course, would it make a difference in what you predicted for which math course they were placed in?
• Do you think taking a course caused a student to place
| **MP.1** | **Make sense of problems and persevere in solving them.** | Students choose an appropriate method of analysis based on problem context. They consider how the data were collected and how data can be summarized to answer statistical questions. Students select a graphical display appropriate to the problem context. They select numerical summaries appropriate to the shape of the data distribution. Students use multiple representations and numerical summaries and then determine the most appropriate representation and summary for a given data distribution. | **Eureka Math:** Module 2 Lesson 9 |
In Topic D, students analyze relationships between two quantitative variables using scatterplots and by summarizing linear relationships using the least squares regression line. Models are proposed based on an understanding of the equations representing the models and the observed pattern in the scatter plot. Students calculate and analyze residuals based on an interpretation of residuals as prediction errors.

**Big Idea:**
- Synthesizing information from multiple sets of data results in evidence-based interpretation.
- A relationship between two numerical variables can be described as a linear or nonlinear relationship.
- Models can be used to answer questions about how two variables are related.

**Essential Questions:**
- Why are multiple sets of data used?
- How can a graph be used to make a prediction? How can a model be used to make a prediction?
- Why does correlation not imply causation?
- How do the patterns in a residual plot show us whether a linear model is a good fit for the data?
- How are residuals used to evaluate the accuracy of predictions based on the least-squares line?

**Vocabulary**
- Residuals, the least-squares line (best-fit line), residual plot, correlation coefficient, causation

**Assessments**
- Live Binders/Galileo: Topic D Assessment

<table>
<thead>
<tr>
<th>Standard</th>
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<th>Resources</th>
</tr>
</thead>
<tbody>
<tr>
<td>S.ID.B.6</td>
<td>B. Summarize, represent, and interpret data on a single count or measurement variable</td>
<td>Explanation: This standard could be incorporated in Unit 2. Students may use spreadsheets, graphing calculators, and statistical software to represent data, describe how the variables are related, fit functions to data, perform regressions, and calculate residuals. This standard builds on students’ work from 8th grade with bivariate data and its relationship. Previous studies of relationships primarily focused on linear models. In Algebra I, students take a more sophisticated look at using a linear function to model the relationship between two numerical variables. In addition to fitting a line to data, students assess how well the model fits by analyzing residuals.</td>
<td>Eureka Math: Module 2 Lesson 12-17, 20 Big Ideas: a. pp. 195, 198, 200, 201-208 b. pp. 202, 203, 206-208 c. 195, 198, 200 (4.4), 201-208 Illuminations: Traveling Distances</td>
</tr>
</tbody>
</table>
This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.

Part a of this standard is taught in Algebra I and Algebra II. In Algebra I, tasks have a real-world context and exponential functions are limited to those with domains in the integers.

Examples S.ID.C.6a:

- In an experiment, 300 pennies were shaken in a cup and poured onto a table. Any penny ‘heads up’ was removed. The remaining pennies were returned to the cup and the process was repeated. The results of the experiment are shown below.

<table>
<thead>
<tr>
<th># of rolls</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td># Pennies</td>
<td>300</td>
<td>164</td>
<td>100</td>
<td>46</td>
<td>20</td>
<td>8</td>
</tr>
</tbody>
</table>

Write a function rule suggested by the context and determine how well it fits the data.

- The rule suggested by the context is $300(0.5)^x$ since the probability of the penny remaining is 50%.

- Which of the following equations best models the (babysitting time, money earned) data?

  $y = x \quad y = \frac{6}{5}x + 2 \quad y = \frac{3}{2}x + 4 \quad y = \frac{1}{4}x + 4$
Informally assess the fit of a function by plotting and analyzing residuals.

A residual is the difference between the actual y-value and the predicted y-value \( (y - \hat{y}) \), which is a measure of the error in prediction. (Note: \( \hat{y} \) is the symbol for the predicted y-value for a given x-value.) A residual is represented on the graph of the data by the vertical distance between a data point and the graph of the function.

Example S.JD.C.6b:
- Lisa lights a candle and records its height in inches every hour. The linear functions \( h(t) = -1.7t + 20 \) has been suggested as a good fit for the data. Use a residual plot to determine the goodness of fit of the function for the data provided in the table.
Measure the wrist and neck size of each person in your class and make a scatterplot. Find the least squares regression line. Calculate and interpret the correlation coefficient for this linear regression model. Graph the residuals and evaluate the fit of the linear equations.

Fit a linear function for a scatter plot that suggests a linear association.

Example S.ID.6c:

The data below gives the number of miles driven and the advertised price for 11 used models of a particular car from 2002 to 2006.

a. Use your calculator to make a scatter plot of the data.
b. Use your calculator to find the correlation coefficient for the data above. Describe what the correlation means in regards to the data.
c. Use your calculator to find an appropriate linear function to model the relationship between miles driven and price for these cars.
d. How do you know that this is the best-fit model?

<table>
<thead>
<tr>
<th>Time (hrs.)</th>
<th>Height (in.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>20</td>
</tr>
<tr>
<td>1</td>
<td>18.3</td>
</tr>
<tr>
<td>2</td>
<td>16.6</td>
</tr>
<tr>
<td>3</td>
<td>14.9</td>
</tr>
<tr>
<td>4</td>
<td>13.2</td>
</tr>
<tr>
<td>5</td>
<td>11.5</td>
</tr>
<tr>
<td>7</td>
<td>8.1</td>
</tr>
<tr>
<td>9</td>
<td>4.7</td>
</tr>
<tr>
<td>10</td>
<td>3</td>
</tr>
</tbody>
</table>
C. Interpret linear models

Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.

_This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential._

<table>
<thead>
<tr>
<th>Miles driven (In thousands)</th>
<th>Price (In dollars)</th>
</tr>
</thead>
<tbody>
<tr>
<td>22</td>
<td>17,998</td>
</tr>
<tr>
<td>29</td>
<td>16,450</td>
</tr>
<tr>
<td>35</td>
<td>14,998</td>
</tr>
<tr>
<td>39</td>
<td>13,998</td>
</tr>
<tr>
<td>45</td>
<td>14,599</td>
</tr>
<tr>
<td>49</td>
<td>14,988</td>
</tr>
<tr>
<td>55</td>
<td>13,599</td>
</tr>
<tr>
<td>56</td>
<td>14,599</td>
</tr>
<tr>
<td>69</td>
<td>11,998</td>
</tr>
<tr>
<td>70</td>
<td>14,450</td>
</tr>
<tr>
<td>86</td>
<td>10,998</td>
</tr>
</tbody>
</table>

Explanation:

_This standard could be incorporated in Unit 2._

Students may use spreadsheets or graphing calculators to create representations of data sets and create linear models.

The line that has a smaller sum of squared residuals for a data set than any other line is called the _least-squares line_. This line can also be called the _best-fit line_ or the _line of best fit_ (or _regression line_).

Once you have found the equation of the least-squares line, the values of the slope and _y_-intercept of the line often reveals something interesting about the relationship you are modeling. The slope of the least-squares line is the change in the predicted value of the _y_ variable associated with an increase of one in the value of the _x_-variable.

Example:

- Data was collected of the weight of a male white laboratory rat for the first 25 weeks after its birth. A scatterplot of the rat’s weight (in grams) and the time since birth (in weeks) shows a fairly strong, positive linear relationship. The linear
regression equation \( W = 100 + 40t \) (where \( W \) = weight in grams and \( t \) = number of weeks since birth) models the data fairly well.

a. What is the slope of the linear regression equation? Explain what it means in context.

b. What is the y-intercept of the linear regression equation? Explain what it means in context.

- Lisa lights a candle and records its height in inches every hour. The results recorded as \((\text{time, height})\) are \((0, 20), (1, 18.3), (2, 16.6), (3, 14.9), (4, 13.2), (5, 11.5), (7, 8.1), (9, 4.7), \) and \((10, 3)\). Express the candle’s height \((h)\) as a function of time \((t)\) and state the meaning of the slope and the intercept in terms of the burning candle.

Solution:

\[ h = -1.7t + 20 \]

Slope: The candle’s height decreases by 1.7 inches for each hour it is burning.

Intercept: Before the candle begins to burn, its height is 20 inches.

- Give an interpretation of the slope of the least-squares line \( y = 25.3 + 3.66x \) for predicting height from shoe size for adult men.

The slope is 3.66; for every one-inch increase in a man’s shoe length, you would estimate an increase of 3.66 inches in height.

The y-intercept of a line is the predicted value of \( y \) when \( x \) equals zero. When using a line as a model for the relationship between two numerical variables, it often does not make sense to interpret the y-intercept because an x-value of zero may not make any sense.

Explain why it does not make sense to interpret the y-intercept of 25.3 as the predicted height for an adult male.
whose shoe length is zero.

*The y-intercept is (0, 25.3). Since x represents shoe length, it is impossible for the shoe length to be 0 inches when a man is 25.3 inches tall.*

<table>
<thead>
<tr>
<th>S.ID.C.8</th>
<th>C. Interpret linear models</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Compute (using technology) and interpret the correlation coefficient of a linear fit.</td>
</tr>
<tr>
<td></td>
<td><em>This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.</em></td>
</tr>
</tbody>
</table>

**Explanation:**

Students may use spreadsheets, graphing calculators, and statistical software to represent data, describe how the variables are related, fit functions to data, perform regressions, and calculate residuals and correlation coefficients.

Build on students’ work with linear relationships in eighth grade and introduce the correlation coefficient. The focus here is on the computation and interpretation of the correlation coefficient as a measure of how well the data fit the relationship.

The correlation coefficient is a number between −1 and +1 (including −1 and +1) that measures the strength and direction of a linear relationship between two quantities in a set of data. The correlation coefficient is denoted by the letter r.

The magnitude (absolute value) of r indicates how closely the data points fit a linear pattern. If r = 1, the points all fall on a line. The closer |r| is to 1, the stronger the correlation. The closer |r| is to zero, the weaker the correlation. The sign of r indicates the direction of the relationship – positive or negative.

**Note:** Students often misinterpret the correlation coefficient to mean that a given linear model is a good or bad fit for the data especially since it is often provided when completing a linear regression on the calculator. The correlation coefficient, r, measures the strength and the direction of a linear relationship between two variables. It can be calculated without a linear model. It is independent of the linear model and thus does not describe the fit of the model. Students should use residuals to assess the fit of the function.

**Examples:**

<table>
<thead>
<tr>
<th>Eureka Math:</th>
<th>Module 2 Lesson 19,20</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Big Ideas:</strong></td>
<td>pp. 203, 204, 206-208</td>
</tr>
<tr>
<td><strong>Other:</strong></td>
<td></td>
</tr>
<tr>
<td>Illuminations – <em>Traveling Distances</em></td>
<td></td>
</tr>
<tr>
<td>Illuminations – <em>Bathtub Water Levels</em></td>
<td></td>
</tr>
<tr>
<td>Illuminations – <em>Automobile Mileage: Year vs Mileage</em></td>
<td></td>
</tr>
<tr>
<td>Texas Instruments:</td>
<td></td>
</tr>
<tr>
<td><em>Influencing Regression</em></td>
<td></td>
</tr>
<tr>
<td><em>Investigating correlation</em></td>
<td></td>
</tr>
<tr>
<td><em>Monopoly and Regression</em></td>
<td></td>
</tr>
</tbody>
</table>
• Collect height, shoe-size, and wrist circumference data for each student. Determine the best way to display the data. Answer the following questions: Is there a correlation between any two of the three indicators? Is there a correlation between all three indicators? What patterns and trends are apparent in the data? What inferences can be made from the data?

• The correlation coefficient of a given data set is 0.97. List three specific things this tells you about the data.

• Is there a connection between the slope of the least-squares line and the value of the correlation coefficient or $r$? If yes, what is the connection?

  There is a connection regarding the sign of $r$ (the correlation coefficient) and the sign of the slope if there is a linear relationship. If the least-squares line is increasing, the slope is positive and the value of the correlation coefficient, or $r$, is positive. If the least-squares line is decreasing, then the slope is negative and the value of the correlation coefficient is negative.

• Why is it important to know if a relationship is strong or weak?

  If a relationship is strong, then the data are close to the line, and the equation of the line can be used to predict values. If the relationship is weak, then the equation cannot be used as easily to predict values.

Given a set of data:

• Based on the scatter plot, do you think that the value of the correlation coefficient between ______ and _____ will be positive or negative? Explain why you made this choice.

• Based on the scatter plot, estimate the value of the correlation coefficient between ______ and ______.

• Calculate the value of the correlation coefficient between ______ and ______. Round to the nearest hundredth. Interpret this value.

---

<table>
<thead>
<tr>
<th>S.ID.C.9</th>
<th>C. Interpret linear models</th>
<th>Explanation: Some data leads observers to believe that there is a cause and effect relationship when a strong relationship is observed. Students should</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Distinguish between correlation and causation.</td>
<td>Eureka Math: Module 2 Lesson 19,20</td>
</tr>
</tbody>
</table>
This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.

To be careful not to assume that correlation implies causation. The determination that one thing causes another requires a controlled randomized experiment. It is sometimes tempting to conclude that if there is a strong linear relationship between two variables that one variable is causing the value of the other variable to increase or decrease. But you should avoid making this mistake. When there is a strong linear relationship, it means that the two variables tend to vary together in a predictable way, which might be due to something other than a cause-and-effect relationship.

Example:

- The value of the correlation coefficient between sodium content and number of calories for the fast food items in an example was $r = 0.79$, indicating a strong positive relationship. This means that the items with higher sodium content tend to have a higher number of calories. But the high number of calories is not caused by the high sodium content. In fact sodium does not have any calories. What may be happening is that food items with high sodium content also may be the items that are high in sugar and/or fat, and this is the reason for the higher number of calories in these items.

- Similarly, there is a strong positive correlation between shoe size and reading ability in children. But it would be silly to think that having big feet causes children to read better. It just means that the two variables vary together in a predictable way. Can you think of a reason that might explain why children with larger feet also tend to score higher on reading tests?

MP.1 Make sense of problems and persevere in solving them.

Students choose an appropriate method of analysis based on problem context. They consider how the data were collected and how data can be summarized to answer statistical questions. Students select a graphical display appropriate to the problem context. They select numerical summaries appropriate to the shape of the data distribution. Students use multiple representations and numerical summaries and then determine the most appropriate representation.

Big Ideas:
Pg. 205 Ex 5
Pg. 207 #21-24

Eureka Math:
Module 2 Lesson 19
| MP.2 | Reason abstractly and quantitatively. | Students pose statistical questions and reason about how to collect and interpret data in order to answer these questions. Students form summaries of data using graphs, two-way tables, and other representations that are appropriate for a given context and the statistical question they are trying to answer. Students reason about whether two variables are associated by considering conditional relative frequencies. | Eureka Math:  
Module 2 Lesson 15  
Module 2 Lesson 17  
Module 2 Lesson 18 |
| MP.3 | Construct viable arguments and critique the reasoning of others. | Students examine the shape, center, and variability of a data distribution and use characteristics of the data distribution to communicate the answer to a statistical question in the form of a poster presentation. Students also have an opportunity to critique poster presentations made by other students. | Eureka Math:  
Module 2 Lesson 18  
Module 2 Lesson 20 |
| MP.4 | Model with mathematics. | Students construct and interpret two-way tables to summarize bivariate categorical data. Students graph bivariate numerical data using a scatterplot and propose a linear, exponential, quadratic, or other model to describe the relationship between two numerical variables. Students use residuals and residual plots to assess if a linear model is an appropriate way to summarize the relationship between two numerical variables. | Eureka Math:  
Module 2 Lesson 12  
Module 2 Lesson 13  
Module 2 Lesson 14  
Module 2 Lesson 15  
Module 2 Lesson 16 |
| MP.5 | Use appropriate tools strategically. | Students visualize data distributions and relationships between numerical variables using graphing software. They select and analyze models that are fit using appropriate technology to determine whether or not the model is appropriate. Students use visual representations of data distributions from technology to answer statistical questions. | Eureka Math:  
Module 2 Lesson 17 |
| MP.6 | Attend to precision. | Students interpret and communicate conclusions in context based on graphical and numerical data summaries. Students use statistical terminology appropriately. | Eureka Math:  
Module 2 Lesson 16 |
In Grade 8, students use functions for the first time to construct a function to model a linear relationship between two quantities (8.F.4) and to describe qualitatively the functional relationship between two quantities by analyzing a graph (8.F.5). In the first four modules of Grade 9, students learn to create and apply linear, quadratic, and exponential functions, in addition to square and cube root functions (F-IF.C.7). In Module 5, they synthesize what they have learned during the year by selecting the correct function type in a series of modeling problems without the benefit of a module or lesson title that includes function type to guide them in their choices. This supports the CCSS requirement that students use the modeling cycle, in the beginning of which they must formulate a strategy. Skills and knowledge from the previous modules will support the requirements of this module, including writing, rewriting, comparing, and graphing functions (F-IF.C.7, F-IF.C.8, F-IF.C.9) and interpretation of the parameters of an equation (F-LE.B.5). They also draw on their study of statistics in Module 2, using graphs and functions to model a context presented with data and/or tables of values (S-ID.B.6). In this module, we use the modeling cycle (see page 72 of the CCSS) as the organizing structure, rather than function type.

Topic A focuses on the skills inherent in the modeling process: representing graphs, data sets, or verbal descriptions using explicit expressions (F-BF.A.1a) when presented in graphic form in Lesson 1, as data in Lesson 2, or as a verbal description of a contextual situation in Lesson 3. They recognize the function type associated with the problem (F-LE.A.1b, F-LE.A.1c) and match to or create 1- and 2-variable equations (A-CED.A.1, A-CED.2) to model a context presented graphically, as a data set, or as a description (F-LE.A.2). Function types include linear, quadratic, exponential, square root, cube root, absolute value, and other piecewise functions. Students interpret features of a graph in order to write an equation that can be used to model it and the function (F-IF.B.4, F-BF.A.1) and relate the domain to both representations (F-IF.B.5). This topic focuses on the skills needed to complete the modeling cycle and sometimes uses purely mathematical models, sometimes real-world contexts.

Big Idea:
- Graphs are used to represent a function and to model a context.
- Identifying a parent function and thinking of the transformation of the parent function to the graph of the function can help with creating the analytical representation of the function.

Essential Questions:
- When presented with a graph, what is the most important key feature that will help one recognize they type of function it represents?
- Which graphs have a minimum/maximum value?
- Which graphs have domain restrictions?
- Which of the parent functions are transformations of other parent functions?
- How is one able to recognize the function if the graph is a transformation of the parent function?
- How would one know which function to use to model a word problem?

Vocabulary
- Analytic model, descriptive model, (function, range, parent function, linear function, quadratic function, exponential function, average rate of change, cube root function, square root function, end behavior, recursive process, piecewise defined function, parameter, arithmetic sequence, geometric sequence, first differences, second differences, analytical model)

Assessments
- Galileo: Foundational Skills Assessment for Module 5; topic A assessment

<table>
<thead>
<tr>
<th>Standard</th>
<th>AZ College and Career Readiness Standards</th>
<th>explanations &amp; Examples</th>
<th>Resources</th>
</tr>
</thead>
<tbody>
<tr>
<td>N.Q.A.2</td>
<td>A. Reason quantitatively and use units to solve problems.</td>
<td>Explanation: Determine and interpret appropriate quantities when using descriptive modeling.</td>
<td>Eureka Math: Module 5 Lesson 1 - 3</td>
</tr>
</tbody>
</table>

6/12/2016
| A.CED.A.1 | A. Create equations that describe numbers or relationships
Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.
This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.

This standard is taught in Algebra I and Algebra II. In Algebra I, the standard will be assessed by ensuring that some modeling tasks (involving Algebra I content or securely held content from grades 6-8) require the student to create a quantity of interest in the situation being described.

Examples: (refer to the examples from Unit 1)
| Explanation:
This standard is taught in Algebra I and Algebra II. In Algebra I, the standard will be assessed by ensuring that some modeling tasks (involving Algebra I content or securely held content from grades 6-8) require the student to create a quantity of interest in the situation being described.

Examples: (refer to the examples from Unit 1)
| Eureka Math: Module 5 Lesson 1 - 3 |
| A.CED.A.2 | A. Create equations that describe numbers or relationships
Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.

This standard is taught in Algebra I and Algebra II. In Algebra I, tasks are limited to linear, quadratic or exponential equations with integer exponents. Students recognize when a problem can be modeled with an equation or inequality and are able to write the equation or inequality. Students create, select, and use graphical, tabular and/or algebraic representations to solve the problem.

Equations can represent real world and mathematical problems. Include equations and inequalities that arise when comparing the values of two different functions, such as one describing linear growth and one describing exponential growth.

Examples: (Refer to examples listed in Units 1,2 and 3)
| Explanation:
This standard is taught in Algebra I and Algebra II. In Algebra I, tasks are limited to linear, quadratic or exponential equations with integer exponents. Students recognize when a problem can be modeled with an equation or inequality and are able to write the equation or inequality. Students create, select, and use graphical, tabular and/or algebraic representations to solve the problem.

Equations can represent real world and mathematical problems. Include equations and inequalities that arise when comparing the values of two different functions, such as one describing linear growth and one describing exponential growth.

Examples: (Refer to examples from Unit 1 and Unit 3 in addition to the examples below)
<p>| Eureka Math: Module 5 Lesson 1 - 3 |</p>
<table>
<thead>
<tr>
<th>F.IF.B.4</th>
<th>B. Interpret functions that arise in applications in terms of the context</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. <strong>Key features include:</strong> intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. <strong>This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.</strong></td>
</tr>
<tr>
<td></td>
<td><strong>Explanation:</strong> This standard is taught in Algebra I and Algebra II. In Algebra I, tasks have a real-world context and they are limited to linear functions, quadratic functions, square-root functions, cube-root functions, piecewise functions (including step functions and absolute-value functions), and exponential functions with domains in the integers. Some functions “tell a story” hence the portion of the standard that has students sketching graphs given a verbal description. Students should have experience with a wide variety of these types of functions and be flexible in thinking about functions and key features using tables and graphs. Examples of these can be found at <a href="http://graphingstories.com">http://graphingstories.com</a>. <strong>Examples:</strong> (Refer to examples in Units 2 and 3)</td>
</tr>
<tr>
<td>Eureka Math: Module 5 Lesson 1 - 3</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>F.IF.B.5</th>
<th>B. Interpret functions that arise in applications in terms of the context</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Relate the domain of a function to its graph and, where</td>
</tr>
<tr>
<td></td>
<td><strong>Explanation:</strong> Students explain the domain of a function from a given context. Students may explain orally, or in written format, the existing relationships. Given the graph of a function, determine the practical</td>
</tr>
<tr>
<td>Eureka Math: Module 5 Lesson 1 - 3</td>
<td></td>
</tr>
</tbody>
</table>
For example, if the function \( h(n) \) gives the number of person-hours it takes to assemble \( n \) engines in a factory, then the positive integers would be an appropriate domain for the function.

This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.

**Example:** (Refer to the examples in Unit 2)

<table>
<thead>
<tr>
<th>Standard</th>
<th>Description</th>
<th>Explanation</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>F.IF.B.6</td>
<td>B. Interpret functions that arise in applications in terms of the context</td>
<td>Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.</td>
<td>Students were first introduced to the concept of rate of change in grade 6 and continued exploration of the concept throughout grades 7 and 8. In Algebra I, students will extend this knowledge to non-linear functions. This standard is taught in Algebra I and Algebra II. In Algebra I, tasks have a real-world context and are limited to linear functions, quadratic functions, square root functions, cube root functions, piecewise-defined functions (including step functions and absolute value functions), and exponential functions with domains in the integers. Examples: (Refer to the examples in Unit 2)</td>
</tr>
<tr>
<td>F.BF.A.1a</td>
<td>Build a function that models a relationship between two quantities</td>
<td>Write a function that describes a relationship between two quantities. a. Determine an explicit expression, a recursive process, or steps for calculation from a context. This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model.</td>
<td>Tasks have a real-world context. In Algebra I, tasks are limited to linear functions, quadratic functions, and exponential functions with domains in the integers. Students will analyze a given problem to determine the function expressed by identifying patterns in the function’s rate of change. They will specify intervals of increase, decrease, constancy, and, if possible, relate them to the function’s description in words or graphically. Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to model functions.</td>
</tr>
</tbody>
</table>
and use the model to solve problems are essential.

<table>
<thead>
<tr>
<th>F.LE.A.1bc</th>
<th><strong>A. Construct and compare linear, quadratic, and exponential models and solve problems</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Distinguish between situations that can be modeled with linear functions and with exponential functions.</td>
</tr>
<tr>
<td></td>
<td>b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.</td>
</tr>
<tr>
<td></td>
<td>c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.</td>
</tr>
</tbody>
</table>

*This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.*

**Explanation:**
Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to model and compare linear and exponential functions.

**Students recognize situations where one quantity changes at a constant rate per unit interval relative to another.**

**Examples:**
- A cell phone company has three plans. Graph the equation for each plan, and analyze the change as the number of minutes used increases. When is it beneficial to enroll in each of the three plans?
  - $59.95/month for 700 minutes and $0.25 for each additional minute,
  - $39.95/month for 400 minutes and $0.15 for each additional minute,
  - $89.95/month for 1,400 minutes and $0.05 for each additional minute

**Students recognize situations where one quantity changes another changes by a constant percent rate.**

When working with symbolic form of the relationship, if the equation can be rewritten in the form \( y = (1 \pm r)^t \), then the relationship is exponential and the constant percent rate per unit interval is \( r \).

When working with a table or graph, either write the corresponding equation and see if it is exponential or locate at least two pairs of points and calculate the percent rate of change for each set of points. If these percent rates are the same, the function is exponential. If the percent rates are not all the same, the function is not exponential.

**Examples:**
- A couple wants to buy a house in five years. They need to
The down payment of $8,000. They deposit $1,000 in a bank account earning 3.25% interest, compounded quarterly. How long will they need to save in order to meet their goal?

- Carbon 14 is a common form of carbon which decays exponentially over time. The half-life of Carbon 14, that is the amount of time it takes for half of any amount of Carbon 14 to decay, is approximately 5730 years. Suppose we have a plant fossil and that the plant, at the time it died, contained 10 micrograms of Carbon 14 (one microgram is equal to one millionth of a gram).
- Using this information, make a table to calculate how much Carbon 14 remains in the fossilized plant after \( n \) number of half-lives.
- How much carbon remains in the fossilized plant after 2865 years? Explain how you know.
- When is there one microgram of Carbon 14 remaining in the fossil?

<table>
<thead>
<tr>
<th>F.LE.A.2</th>
<th>A. Construct and compare linear, quadratic, and exponential models and solve problems</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).</td>
</tr>
<tr>
<td></td>
<td>This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>MP.1</th>
<th>Make sense of problems and persevere in solving them.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. In Module 5,</td>
</tr>
</tbody>
</table>

**Explanation:**
This standard is taught in Algebra I and Algebra II. In Algebra I, tasks are limited to constructing linear and exponential functions in simple context (not multi-step).

While working with arithmetic sequences, make the connection to linear functions, introduced in 8th grade. Geometric sequences are included as contrast to foreshadow work with exponential functions later in the course.

Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to construct linear and exponential functions.

**Examples:** refer to examples from Unit 2

**Eureka Math:**
Module 5 Lesson 1 - 3

**Eureka Math:**
Module 5 Lesson 1
Module 5 Lesson 2
students make sense of the problem by analyzing the critical components of the problem, presented as a verbal description, a data set, or a graph and persevere in writing the appropriate function that describes the relationship between two quantities. Then, they interpret the function in the context.

<table>
<thead>
<tr>
<th>MP.3</th>
<th>Construct viable arguments and critique the reasoning of others.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>MP.4</th>
<th>Model with mathematics.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In this module, students create a function from a contextual situation described verbally, create a graph of their function, interpret key features of both the function and the graph in the terms of the context, and answer questions related to the function and its graph. They also create a function from a data set based on a contextual situation. In Topic B, students use the full modeling cycle with functions presented mathematically or in a context, including linear, quadratic, and exponential. They explain their mathematical thinking in writing and/or by using appropriate tools, such as graph paper, graphing calculator, or computer software.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>MP.5</th>
<th>Use appropriate tools strategically.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. Throughout the entire module students must decide whether or not to use a tool to help find solutions. They must graph functions that are sometimes difficult to sketch (e.g., cube root and square root) and sometimes are required to perform procedures that can be tedious, and sometimes distract from the mathematical thinking, when performed without technology (e.g., completing the square with non-integer coefficients).</td>
</tr>
</tbody>
</table>

Eureka Math: Module 5 Lesson 2

Eureka Math: Module 5 Lesson 1

Eureka Math: Module 5 Lesson 3
In these cases, students must decide whether to use a tool to help with the calculation or graph so they can better analyze the model. Students should have access to a graphing calculator for use on the module assessment.

<table>
<thead>
<tr>
<th>MP.8</th>
<th>Look for and express regularity in repeated reasoning.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.</td>
</tr>
</tbody>
</table>

Eureka Math: Module 5 Lesson 2
### Big Idea:
- Data plots and other visual displays of data can help us determine the function type that appears to be the best fit for the data.
- The full modeling cycle is used to interpret the function and its graph, compute for the rate of change over an interval and attend to precision to solve real-world problems in the context of population growth and decay and other problems in geometric sequence or forms of linear, exponential, and quadratic functions.

### Essential Questions:
- Why would one want to represent a graph of a function in analytical form?
- Why would one want to represent a graph as a table of values?

### Vocabulary
Analytic model, descriptive model, (function, range, parent function, linear function, quadratic function, exponential function, average rate of change, cube root function, square root function, end behavior, recursive process, piecewise defined function, parameter, arithmetic sequence, geometric sequence, first differences, second differences, analytical model)

### Assessment
Galileo: Topic B Assessment

### Standard | AZ College and Career Readiness Standards | Explanations & Examples | Comments
--- | --- | --- | ---
N.Q.A.2 | A. Reason qualitatively and units to solve problems
Define appropriate quantities for the purpose of descriptive modeling.

**This is a modeling standard which means students choose and use appropriate mathematics to analyze**

Explanation:
Determine and interpret appropriate quantities when using descriptive modeling.

This standard is taught in Algebra I and Algebra II. In Algebra I, the standard will be assessed by ensuring that some modeling tasks (involving Algebra I content or securely held content from grades 6-8) require the student to create a quantity of interest in the situation.

Eureka Math: Module 5 Lesson 4 - 9

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situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.

**Being described.**

Examples: (refer to the examples from Unit 1)

<table>
<thead>
<tr>
<th>N.Q.A.3</th>
<th>A. Reason qualitatively and units to solve problems</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.</td>
</tr>
<tr>
<td></td>
<td><em>This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.</em></td>
</tr>
<tr>
<td></td>
<td>Explanation: The margin of error and tolerance limit varies according to the measure, tool used, and context.</td>
</tr>
<tr>
<td></td>
<td>Examples: (Refer to the examples from Unit 1)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>A.CED.A.1</th>
<th>A. Create equations that describe numbers or relationships</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Create equations and inequalities in one variable and use them to solve problems. <em>Include equations arising from linear and quadratic functions, and simple rational and exponential functions.</em></td>
</tr>
<tr>
<td></td>
<td><em>This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.</em></td>
</tr>
<tr>
<td></td>
<td>Explanation: This standard is taught in Algebra I and Algebra II. In Algebra I, tasks are limited to linear, quadratic or exponential equations with integer exponents. Students recognize when a problem can be modeled with an equation or inequality and are able to write the equation or inequality. Students create, select, and use graphical, tabular and/or algebraic representations to solve the problem.</td>
</tr>
<tr>
<td></td>
<td>Equations can represent real world and mathematical problems. Include equations and inequalities that arise when comparing the values of two different functions, such as one describing linear growth and one describing exponential growth.</td>
</tr>
<tr>
<td></td>
<td>Examples: (Refer to examples listed in Units 1,2 and 3)</td>
</tr>
</tbody>
</table>

Eureka Math: Module 5 Lesson 4 - 9
Alex designed a new snowboard. He wants to market it and make a profit. The total initial cost for manufacturing set-up, advertising, etc. is $500,000 and the materials to make the snowboards cost $100 per board.

The demand function for selling a similar snowboard is: \( D(p) = 50,000 - 100p \), where \( p \) = selling price of each snowboard.

a. Write an expression for each of the following. Let \( p \) represent the selling price:

   Demand Function [number of units that will sell]
   
   This is given as \( 50,000 - 100p \).

   Revenue [number of units that will sell, price per unit, \( p \)]
   
   \( (50,000 - 100p)p = 50,000p - 100p^2 \)

   Total Cost [cost for producing the snowboards]
   
   This is the total overhead costs, plus the cost per snowboard times the number of snowboards:
   
   \( 500,000 + 100(50,000 - 100p) \)
   
   \( 500,000 + 5,000,000 - 10,000p \)
   
   \( 5,500,000 - 10,000p \)
A.CED.A.2  

A. Create equations that describe numbers or relationships

Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.

Explanation:
This standard is taught in Algebra I and Algebra II. In Algebra I, students create equations in two variables for linear, exponential and quadratic contextual situations. Limit exponential situations to only ones involving integer input values.

Examples: (Refer to examples from Units 1 & 3 in addition to the examples below)

- A culture of bacteria doubles every 2 hours.
  a. Explain how this situation can be modeled with a sequence.
  
  To find the next number of bacteria, you multiply the previous number by 2. This situation can be represented by a geometric sequence. There will be a common ratio between each term of the sequence.
  
  b. If there are 500 bacteria at the beginning, how many bacteria will there be after 24 hours?
  
  Since the bacteria splits (reproduces) every two hours, then n represents 12 splits in the 24 hour period.
  
  \[ f(n) = \text{initial amount} \times (\text{common ratio})^{n-1} \]  
  Or, in terms of the context: the 500 bacteria cells will double 11 times in 24 hours.
  
  \[ f(n) = 500(2)^{11-1} = 500(2)^{10} = 1,024,000 \text{ bacteria} \]

- Maria invested $10,000 in the stock market. Unfortunately, the value of her investment has been dropping at an average rate of 3% each year.
  a. Write the function that best models the situation.
  
  \[ f(n) = 10,000(1 - 0.03)^n \text{ or } 10,000(0.97)^n \]
  where \( n \) = number of years since the initial investment

F.IF.B.4  

B. Interpret functions that arise in applications in terms of the context

For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing

Explanation:
This standard is taught in Algebra I and Algebra II. In Algebra I, tasks have a real-world context and they are limited to linear functions, quadratic functions, square-root functions, cube-root functions, piecewise functions (including step functions and absolute-value functions), and exponential functions with domains in the integers.

Eureka Math: Module 5 Lesson 4 - 9
### F.IF.B.5

**B. Interpret functions that arise in applications in terms of the context**

Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function $h(n)$ gives the number of person-hours it takes to assemble $n$ engines in a factory, then the positive integers would be an appropriate domain for the function.

*This is a modeling standard which means students choose and use appropriate mathematics to analyze situations.*

**Explanation:**

Students explain the domain of a function from a given context. Students may explain orally, or in written format, the existing relationships. Given the graph of a function, determine the practical domain of the function as it relates to the numerical relationship it describes.

**Examples:** (Refer to the examples in Unit 2 in addition to the example below)

- A rocket is launched from 180 feet above the ground at time $t = 0$. The function that models this situation is given by $h = -16t^2 + 96t + 180$, where $t$ is measured in seconds and $h$ is height above the ground measured in feet.
  - What is a reasonable domain restriction for $t$ in this context?
  - Determine the height of the rocket two seconds after it was launched.
  - Determine the maximum height obtained by the rocket.
  - Determine the time when the rocket is 100 feet above the ground.
  - Determine the time at which the rocket hits the ground.
  - How would you refine your answer to the first question based on your response to the second and fifth questions?

---

### Some functions “tell a story” hence the portion of the standard that has students sketching graphs given a verbal description. Students should have experience with a wide variety of these types of functions and be flexible in thinking about functions and key features using tables and graphs. Examples of these can be found at [http://graphingstories.com](http://graphingstories.com)

**Examples:** (Refer to examples in Units 2 and 3 in addition to the example below)

- A rocket is launched from 180 feet above the ground at time $t = 0$. The function that models this situation is given by $h = -16t^2 + 96t + 180$, where $t$ is measured in seconds and $h$ is height above the ground measured in feet.
  - What is a reasonable domain restriction for $t$ in this context?
  - Determine the height of the rocket two seconds after it was launched.
  - Determine the maximum height obtained by the rocket.
  - Determine the time when the rocket is 100 feet above the ground.
  - Determine the time at which the rocket hits the ground.
  - How would you refine your answer to the first question based on your response to the second and fifth questions?
situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.

F.IF.B.6

B. Interpret functions that arise in applications in terms of the context

Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

Explanation:
Students were first introduced to the concept of rate of change in grade 6 and continued exploration of the concept throughout grades 7 and 8. In Algebra I, students will extend this knowledge to non-linear functions.

This standard is taught in Algebra I and Algebra II. In Algebra I, tasks have a real-world context and are limited to linear functions, quadratic functions, square root functions, cube root functions, piecewise-defined functions (including step functions and absolute value.

Eureka Math:
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1. One data point on the graph of \( f \) appears to be \((80, 1000)\). What do you think this point represents in the context of this problem? Explain your reasoning.

In this problem, 80 would represent the speed in mph and 1000 would represent the stopping distance in feet. It doesn’t make sense for a car to be traveling at 1000 mph, much less only take 80 feet to come to a stop.
**F.BF.A.1a**  
**Build a function that models a relationship between two quantities**  
Write a function that describes a relationship between two quantities.  
   a. Determine an explicit expression, a recursive process, or steps for calculation from a context.  

*This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.*

**Example:** (Refer to the examples in Unit 2)

- **Explanation:**  
  This standard is explored further in Topic D. In this topic, it is explored via sequences and exponential growth/decay. The students will analyze a given problem to determine the function expressed by identifying patterns in the function’s rate of change. Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to model functions.

- **Explanation:**  
  This standard is taught in Algebra I and Algebra II. In Algebra I, tasks have a real-world context and are limited to linear functions, quadratic functions, and exponential functions with domains in the integers.

**Examples (refer to the examples from Unit 1 in addition to the examples below):**

- You buy a $10,000 car with an annual interest rate of 6 percent compounded annually and make monthly payments of $250. Express the amount remaining to be paid off as a function of the number of months, using a recursion equation.

- A cup of coffee is initially at a temperature of 93°F. The difference between its temperature and the room temperature of 68°F decreases by 9% each minute. Write a function describing the temperature of the coffee as a function of time.

- The radius of a circular oil slick after $t$ hours is given in feet by $r = 10t^2 - 0.5t$, for $0 \leq t \leq 10$. Find the area of the oil slick as a function of time.

---

**F.LE.A.1bc**  
**A. Construct and compare linear, quadratic, and exponential models and solve problems**  
Distinguish between situations that can be modeled with linear functions and with exponential functions.
   b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

**Explanation:**  
Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to model and compare linear and exponential functions.

**Students recognize situations where one quantity changes at a constant rate per unit interval relative to another.**

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changes at a constant rate per unit interval relative to another.

This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.

<table>
<thead>
<tr>
<th>Examples:</th>
</tr>
</thead>
<tbody>
<tr>
<td>• A cell phone company has three plans. Graph the equation for each plan, and analyze the change as the number of minutes used increases. When is it beneficial to enroll in each of the three plans?</td>
</tr>
<tr>
<td>o $59.95/month for 700 minutes and $0.25 for each additional minute,</td>
</tr>
<tr>
<td>o $39.95/month for 400 minutes and $0.15 for each additional minute,</td>
</tr>
<tr>
<td>o $89.95/month for 1,400 minutes and $0.05 for each additional minute</td>
</tr>
</tbody>
</table>

Students recognize situations where one quantity changes another changes by a constant percent rate.

When working with symbolic form of the relationship, if the equation can be rewritten in the form \( y = a(1 \pm r)^t \), then the relationship is exponential and the constant percent rate per unit interval is \( r \).

When working with a table or graph, either write the corresponding equation and see if it is exponential or locate at least two pairs of points and calculate the percent rate of change for each set of points. If these percent rates are the same, the function is exponential. If the percent rates are not all the same, the function is not exponential.

Examples:

• A couple wants to buy a house in five years. They need to save a down payment of $8,000. They deposit $1,000 in a bank account earning 3.25% interest, compounded quarterly. How long will they need to save in order to meet their goal?

• Carbon 14 is a common form of carbon which decays exponentially over time. The half-life of Carbon 14, that is the amount of time it takes for half of any amount of Carbon 14 to decay, is approximately 5730 years. Suppose
A computer store sells about 200 computers at the price of $1,000 per computer. For each $50 increase in price, about ten fewer computers are sold. How much should the computer store charge per computer in order to maximize their profit?

A couple wants to buy a house in five years. They need to save a down payment of $8,000. They deposit $1,000 in a bank account earning 3.25% interest, compounded quarterly. How much will they need to save each month in order to meet their goal?

Sketch and analyze the graphs of the following two situations. What information can you conclude about the types of growth each type of interest has?

- Lee borrows $9,000 from his mother to buy a car. His mom charges him 5% interest a year, but she does not compound the interest.
- Lee borrows $9,000 from a bank to buy a car. The bank charges 5% interest compounded annually.

Calculate the future value of a given amount of money, with and without technology.

Calculate the present value of a certain amount of money for a given length of time in the future, with and without technology.
A description of a relationship, or two input-output pairs (include reading these from a table).

This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.

While working with arithmetic sequences, make the connection to linear functions, introduced in 8th grade. Geometric sequences are included as contrast to foreshadow work with exponential functions later in the course.

Students may use graphing calculators or programs, spreadsheets, or computer algebra systems to construct linear and exponential functions.

Examples: refer to examples from Unit 2 in addition to the examples below

Lewis' dad put 1,000 dollars in a money market fund when he was 16. Lewis can't touch the money until he is 26, but he gets updates on the balance of his account.

<table>
<thead>
<tr>
<th>Years After Lewis Turns 16</th>
<th>Account Balance in Dollars</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1000</td>
</tr>
<tr>
<td>1</td>
<td>1100</td>
</tr>
<tr>
<td>2</td>
<td>1210</td>
</tr>
<tr>
<td>3</td>
<td>1331</td>
</tr>
<tr>
<td>4</td>
<td>1464</td>
</tr>
</tbody>
</table>
B. Interpret functions that arise in applications in terms of the context

Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require

**Explanation:**

Students were first introduced to the concept of rate of change in grade 6 and continued exploration of the concept throughout grades 7 and 8. In Algebra I, students will extend this knowledge to non-linear functions.

This standard is taught in Algebra I and Algebra II. In Algebra I, tasks have a real-world context and are limited to linear functions, quadratic functions, square root functions, cube root functions, piecewise-defined functions (including step functions and absolute value).
### F.LE.A.3

**A. Construct and compare linear, quadratic, and exponential models and solve problems**

Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

*This is a modeling standard which means students choose and use appropriate mathematics to analyze situations. Thus, contextual situations that require students to determine the correct mathematical model and use the model to solve problems are essential.*

**Explanation:**

Students extend their knowledge of linear functions to compare the characteristics of exponential and quadratic functions; focusing specifically on the value of the quantities. Noting that values of exponential functions are eventually greater than the other function types.

**Examples:** refer to the examples in Unit 2

### MP.1

**Make sense of problems and persevere in solving them.**

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. In Module 5, students make sense of the problem by analyzing the critical components of the problem, presented as a verbal description, a data set, or a graph and persevere in writing the appropriate function that describes the relationship between two quantities. Then, they interpret the function in the context.

### MP.4

**Model with mathematics.**

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In this module, students create a function from a contextual situation described verbally, create a graph of their
function, interpret key features of both the function and the graph in the terms of the context, and answer questions related to the function and its graph. They also create a function from a data set based on a contextual situation. In Topic B, students use the full modeling cycle with functions presented mathematically or in a context, including linear, quadratic, and exponential. They explain their mathematical thinking in writing and/or by using appropriate tools, such as graph paper, graphing calculator, or computer software.

<table>
<thead>
<tr>
<th>MP.6</th>
<th>Attend to precision.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mathematically proficient students try to communicate precisely to others. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure and labeling axes to clarify the correspondence with quantities in a problem. When calculating and reporting quantities in all topics of Module 5 students must choose the appropriate units and use the appropriate level of precision based on the information as it is presented. When graphing they must select an appropriate scale.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>MP.7</th>
<th>Look for and make use of structure.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mathematically proficient students look closely to discern a pattern or structure. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects.</td>
</tr>
</tbody>
</table>