# 6th Grade Math Fourth Quarter

## Unit 5: Geometry
### Topic B: Polygons on the Coordinate Plane

In Unit 3, students used coordinates and absolute value to find distances between points on a coordinate plane (6.NS.C.8). In Topic B, students extend this learning by finding edge lengths of polygons (the distance between two vertices using absolute value) and draw polygons given coordinates (6.G.A.3). From these drawings, students determine the area of polygons on the coordinate plane by composing and decomposing into polygons with known area formulas. Students investigate and calculate the area of polygons on the coordinate plane and also calculate the perimeter. They note that finding perimeter is simply finding the sum of the polygon’s edge lengths (or finding the sum of the distances between vertices). Topic B concludes with students determining distance, perimeter, and area on the coordinate plane in real-world contexts.

### Big Idea:
- Geometry and spatial sense offer ways to envision, to interpret and to reflect on the world around us.
- Area, volume and surface area are measurements that relate to each other and apply to objects and events in our real life experiences.
- Measurement is used to quantify attributes of shapes and objects in order to make sense of our world.

### Essential Questions:
- How does measurement help you solve problems in everyday life?
- How does what we measure influence how we measure?
- How can space be defined through numbers and measurement?
- How does investigating figures help us build our understanding of mathematics?
- What is the relationship with 2-dimensional shapes and our world?
- How can you use area and perimeter formulas to find missing dimensions of plane figures?
- How can a coordinate plane be used to solve measurement problems?

### Vocabulary
- Composite figure, polygon, coordinate plane, vertices, perimeter, area

### AZ College and Career Readiness Standards

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<td>6</td>
<td>G</td>
<td>3</td>
<td>A. Solve real-world and mathematical problems involving area, surface area, and volume.</td>
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Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.

**Explanation:**

Students are given the coordinates of polygons to draw in the coordinate plane. If both x-coordinates are the same (2, -1) and (2, 4), then students recognize that a vertical line has been created and the distance between these coordinates is the distance between -1 and 4, or 5. If both the y-coordinates are the same (-5, 4) and (2, 4), then students recognize that a horizontal line has been created and the distance between these coordinates is the distance between -5 and 2, or 7. Using this understanding, student solve real-world and mathematical problems, including finding the area and perimeter of polygons.
Connections: 6.NS.8; 6-8.RST.7

6.MP.1. Make sense of problems and persevere in solving them.
6.MP.2. Reason abstractly and quantitatively.
6.MP.5. Use appropriate tools strategically.
6.MP.7. Look for and make use of structure.

- Four friends used Google Map to map out their neighborhood. They discovered that their houses form a rectangle. Use a coordinate grid to plot their houses; then answer the questions.
  - How many units away is Henry’s house from Ron’s house?
  - How many units away is Minnie’s house from Henry’s house?
  - Who lives closer to Kim – Ron or Minnie? How do you know?

- On a map, the library is located at (-2, 2), the city hall building is located at (0,2), and the high school is located at (0,0). Represent the locations as points on a coordinate grid with a unit of 1 mile.
  - What is the distance from the library to the city hall building? The distance from the city hall building to the high school? How do you know?
  - What shape is formed by connecting the three locations?
  - The city council is planning to place a city park in this area. How large is the area of the planned park?
Calculate the area of the polygon using two different methods. Write two expressions to represent the two methods and compare the structure of the expressions.

Answers will vary. The following are two possible methods. However, students could also break the shape into two triangles and a rectangle or another correct method.

Method One:

Area of triangle 1 and 4

\[
A = \frac{1}{2}bh
\]

\[A = \frac{1}{2} \cdot 4 \text{ units} \cdot 3 \text{ units} = 6 \text{ units}^2\]

\[A = \frac{1}{2} \cdot 12 \text{ units}^2 = 6 \text{ units}^2\]

Since there are 2, we have a total area of 12 units².

Total area = 12 units² = 24 units² = 36 units²

Method Two:

Area of triangle 2 and 3

\[
A = \frac{1}{2}bh
\]

\[A = \frac{1}{2} \cdot 8 \text{ units} \cdot 6 \text{ units} = 24 \text{ units}^2\]

\[A = \frac{1}{2} \cdot 4 \text{ units}^2 = 4 \text{ units}^2\]

\[A = 3 \text{ units}^2\]

Since there are 2, we have a total area of 14 units².

Total area = 48 units² − 12 units²

Total area = 36 units²

Expressions

\[2 \left( \frac{4}{3} \times \frac{3}{3} \right) \quad \text{or} \quad 8 \left( \frac{3}{3} \right) \]

The first expression shows terms being added together because I separated the hexagon into smaller pieces and had to add their areas back together.

The second expression shows terms being subtracted because I made a larger outside shape, and then had to take away the extra pieces.
Unit 5: Geometry

Topic C: Volume of Right Rectangular Prisms

In Grade 5, students recognized volume as an attribute of solid figures. They measured volume by packing right rectangular prisms with unit cubes and found that determining volume was the same as multiplying the edge lengths of the prism (5.MD.C.3, 5.MD.C.4). Students extend this knowledge to Topic C where they continue packing right rectangular prisms with unit cubes; however, this time the right rectangular prism has fractional lengths (6.G.A.2). Students decompose a one cubic unit prism in order to conceptualize finding the volume of a right rectangular prism with fractional edge lengths using unit cubes. They connect those findings to apply the formula $V = lwh$ and multiply fractional edge lengths (5.NF.B.4). Students extend and apply the volume formula to $V = \text{the area of the base} \times \text{height}$ or simply $V = bh$, where $b$ represents the area of the base. Students explore the bases of right rectangular prisms and find the area of the base first, then multiply by the height. They determine that two formulas can be used to find the volume of a right rectangular prism. Students apply both formulas to application problems. Topic C concludes with real-life application of the volume formula where students extend the notion that volume is additive (5.MD.C.5c) and find the volume of composite solid figures. They apply volume formulas and use their previous experience with solving equations (6.EE.B.7) to find missing volumes and missing dimensions.

Big Idea:

- Geometry and spatial sense offer ways to envision, to interpret and to reflect on the world around us.
- Area, volume and surface area are measurements that relate to each other and apply to objects and events in our real life experiences.
- Properties of 2-dimensional shapes are used in solving problems involving 3-dimensional shapes.

Essential Questions:

- How does what we measure influence how we measure?
- How can space be defined through numbers and measurement?
- What is the relationship between 2-dimensional shapes, 3-dimensional shapes and our world?
- How can you use models to find volume?

Vocabulary

volume, right rectangular prism, composite solid figures, decompose, vertex, face, edge, 3-dimensional figure, solid, unit cube, base area

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AZ College and Career Readiness Standards

A. Solve real-world and mathematical problems involving area, surface area, and volume.

Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = lwh$ and $V = Bh$ to find volumes of right

Explanation:

In 5th grade, students calculated the volume of right rectangular prisms (boxes) using whole number edges. The use of models was emphasized as students worked to derive the formula $V = Bh$ (5.MD.3, 5.MD.4, 5.MD.5) The unit cube was $1 \times 1 \times 1$.

In 6th grade the unit cube will have fractional edge lengths. (ie. $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$) Students find the volume of the right rectangular prism with these

Resources

- Eureka Math: M5 Lesson 11-14
- Big Ideas: Section 8.4
rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.

6.MP.1. Make sense of problems and persevere in solving them.
6.MP.2. Reason abstractly and quantitatively.
6.MP.3. Construct viable arguments and critique the reasoning of others.
6.MP.5. Use appropriate tools strategically.
6.MP.7. Look for and make use of structure.
6.MP.8. Look for and express regularity in repeated reasoning.

Students need multiple opportunities to measure volume by filling rectangular prisms with blocks and looking at the relationship between the total volume and the area of the base. Through these experiences, students derive the volume formula (volume equals the area of the base times the height). Students can explore the connection between filling a box with unit cubes and the volume formula using interactive applets such as the Cubes Tool on NCTM’s Illuminations (http://illuminations.nctm.org/ActivityDetail.aspx?ID=6).

In addition to filling boxes, students can draw diagrams to represent fractional side lengths, connecting with multiplication of fractions. This process is similar to composing and decomposing two dimensional shapes.

Examples:
• The model shows a cubic foot filled with cubic inches. The cubic inches can also be labeled as a fractional cubic unit with dimensions of $\frac{1}{12}$ ft$^3$.

![Diagram of cubic foot divided into smaller units]

• The model shows a rectangular prism with dimensions 3/2 inches, 5/2 inches, and 5/2 inches. Each of the cubic units in the model is $\frac{1}{8}$ in$^3$. Students work with the model to illustrate $3/2 \times 5/2 \times 5/2 = (3 \times 5 \times 5) \times 1/8$. Students reason
that a small cube has volume 1/8 because 8 of them fit in a unit cube.

A company is creating a rectangular prism that must have a volume of 6 ft³. The company also knows that the area of the base must be $\frac{3}{2}$ ft². How can you use what you learned today about volume to determine the height of the rectangular prism?

I know that the volume can be calculated by multiplying the area of the base times the height. So, if I needed the height instead, I would do the opposite. I would divide the volume by the area of the base to determine the height.

\[ V = \text{Area of base} \times \text{height} \]
\[ 6 \text{ ft}^3 = \left( \frac{3}{2} \text{ ft}^2 \right) (h) \]
\[ 6 \text{ ft}^3 = \frac{3}{2} \text{ ft}^2 \times h \]
\[ \frac{3}{2} \text{ ft} = h \]

A special order sand box has been created for children to use as an archeological digging area at the zoo. Determine the volume of the sand box.
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Unit 5: Geometry

Topic D: Nets and Surface Area

Unit 5 concludes with deconstructing the faces of solid figures to determine surface area. Students note the difference between finding the volume of right rectangular prisms and finding the surface area of such prisms. Students build solid figures using nets. They note which nets compose specific solid figures and also understand when nets cannot compose a solid figure. From this knowledge, students deconstruct solid figures into nets to identify the measurement of the solids’ face edges. Students are then prepared to use nets to determine the surface area of solid figures. They find that adding the areas of each face of the solid will result in a combined surface area. Students find that each right rectangular prism has a front, a back, a top, a bottom, and two sides. They determine that surface area is obtained by adding the areas of all the faces. They understand that the front and back of the prism have the same surface area, the top and bottom have the same surface area, and the sides have the same surface area. Thus, students develop the formula \( S.A = 2lw + 2lh + 2wh \) (6.G.A.4). To wrap up the unit, students apply the surface area formula to real-life contexts and distinguish between the need to find surface area or volume within contextual situations.

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<td>Nets, surface area, prism, deconstruct, prism, pyramid</td>
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<td>4</td>
<td>A. Solve real-world and mathematical problems involving area, surface area, and volume. Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these</td>
<td>Explanation: A net is a two-dimensional representation of a three-dimensional figure. Students represent three-dimensional figures whose nets are composed of rectangles and triangles. Students recognize that parallel lines on a net are congruent. Using the dimensions of the individual faces, students calculate the area of each rectangle and/or</td>
<td>Eureka Math: M5 Lesson 15-19a</td>
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techniques in the context of solving real-world and mathematical problems.

6.MP.1. Make sense of problems and persevere in solving them.

6.MP.2. Reason abstractly and quantitatively.

6.MP.3. Construct viable arguments and critique the reasoning of others.


6.MP.5. Use appropriate tools strategically.


6.MP.7. Look for and make use of structure.

6.MP.8. Look for and express regularity in repeated reasoning.

Students construct models and nets of three dimensional figures, describing them by the number of edges, vertices, and faces. Solids include rectangular and triangular prisms. Students are expected to use the net to calculate the surface area.

Students can create nets of 3D figures with specified dimensions using the Dynamic Paper Tool on NCTM’s Illuminations (http://illuminations.nctm.org/ActivityDetail.aspx?ID=205).

Students also describe the types of faces needed to create a three-dimensional figure. Students make and test conjectures by determining what is needed to create a specific three-dimensional figure.

**Examples:**

- Describe the shapes of the faces needed to construct a rectangular pyramid. Cut out the shapes and create a model. Did your faces work? Why or why not?

- Create the net for a given prism or pyramid, and then use the net to calculate the surface area.
Below are the nets for a variety of prisms and pyramids. Classify the solids as prisms or pyramids, and identify the shape of the base(s). Then write the name of the solid.

a. Prism, bases are pentagons
   Pentagonal Prism

b. Pyramid, the base is a rectangle
   Rectangular Pyramid

c. Pyramid, the base is a triangle
   Triangular Pyramid

d. Prism, bases are triangles
   Triangular Prism

This tent is shaped like a triangular prism. It has equilateral bases that measure 5 feet on each side. The tent is 8 feet long. Sketch the net of the tent, and label the edge lengths:

Possible net:
A box needs to be painted. How many square inches will need to be painted to cover every surface?

![Diagram of a box with dimensions: 15 in. x 12 in. x 6 in.]

\[ SA = 2(15 \text{ in.})(12 \text{ in.}) + 2(15 \text{ in.})(6 \text{ in.}) + 2(12 \text{ in.})(6 \text{ in.}) \]
\[ SA = 360 \text{ in}^2 + 180 \text{ in}^2 + 144 \text{ in}^2 \]
\[ SA = 684 \text{ in}^2 \]

A juice box is 4 in. tall, 1 in. wide, and 2 in. long. How much juice fits inside the juice box?

\[ V = 1 \text{ in.} \times 2 \text{ in.} \times 4 \text{ in.} = 8 \text{ in}^3 \]

How did you decide how to solve each problem?

I chose to use surface area to solve the first problem because you would need to know how much area the paint would need to cover. I chose to use volume to solve the second problem because you would need to know how much space is inside the juice box to determine how much juice it can hold.
## 6th Grade Math Fourth Quarter

### Unit 6: Statistics (5 weeks)

#### Topic A: Understanding Data Distribution

In Grade 5, students used bar graphs and line plots to represent data and then solved problems using the information presented in the plots (5.MD.B.2). In this unit, students move from simply representing data into analysis of data. In Topic A, students begin to think and reason statistically, first by recognizing a statistical question as one that can be answered by collecting data (6.SP.A.1). Students learn that the data collected to answer a statistical question has a distribution that is often summarized in terms of center, variability, and shape (6.SP.A.2). Beginning in Topic A, and throughout the unit, students see and represent data distributions using dot plots and histograms (6.SP.B.4).

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<td>• Statistical questions and the answers account for variability in data.</td>
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<td>• The distribution of a data set is described by its center, spread, and overall shape.</td>
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<td>• Measures of center for a numerical set of data are summaries of the values using a single number.</td>
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<td>• Measures of variability describe the variation of the values in the data set using a single number.</td>
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<tr>
<td>• How do we analyze and interpret data sets?</td>
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<td>• When is one data display better than another? How do mathematicians choose to display data in strategic ways?</td>
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<tr>
<td>• When is one statistical measure better than another?</td>
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<tr>
<td>• What is a statistical question?</td>
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<tr>
<td>• Why is it important to look at the shape of data?</td>
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<td>• Why are data displays important?</td>
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<tr>
<td>statistical question, categorical data, numerical data, data, variability, conjecture, dot plots, distribution, spread of data, center of data, frequency tables, frequency histogram, relative frequency histogram, symmetric, skewed, cluster, peak, gap, outlier</td>
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<td>A. Develop understanding of statistical variability. Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. For example, “How old am I?” is not a statistical question, but “How old are the students in my school?” is a statistical question because one anticipates variability in students’ ages. 6.MP.1. Make sense of problems and persevere in</td>
<td>Explanation: Generating statistical questions and looking at the distribution of data to identify measures of center, spread, and overall shape is new in 6th grade. These skills lay the foundation for statistics and probability in 7th and 8th grades. A statistical question is one that collects information that addresses differences in a population. Depending on the question, the type of data gathered can be either categorical or numerical. An example of a</td>
<td>Eureka Math: M6 Lesson 1-5 Big Ideas: Section 9.1</td>
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solving them.  

6.MP.3. Construct viable arguments and critique the reasoning of others.  


categorical question is “What is your favorite type of pizza?” The answers generated by this question will be categories of pizza types such as pepperoni, cheese, or sausage. An example of a numerical question is “How many pencils does each member of our class have in his or her desk?” A variety of numerical answers about the number of pencils would be given by a typical 6th grade class.  

In 6th grade, the focus should be on statistical questions that generate numerical data. Statistics are numerical data relating to a group of individuals; statistics is also the name for the science of collecting, analyzing and interpreting such data. A statistical question anticipates an answer that varies from one individual to the next and is written to account for the variability in the data. Data are the numbers produced in response to a statistical question. Data are frequently collected from surveys or other sources (i.e. documents).  

Once the data is gathered, it can be organized in a table and/or displayed in a graph. The types of graphs to be focused on in 6th grade are dot plots (line plots), histograms, and box plots (box-and-whisker plots.) After organizing the data into tables and/or graphs, students will analyze the data finding measures of center and measures of variation to draw conclusions.  

Students differentiate between statistical questions and those that are not. A statistical question is one that collects information that addresses differences in a population. The question is framed so that the responses will allow for the differences. For example, the question, “How tall am I?” is not a statistical question because there is only one response; however, the question, “How tall are the students in my class?” is a statistical question since the responses anticipates variability by providing a variety of possible anticipated responses that have numerical answers. Questions can result in a narrow or wide range of numerical values. For example, asking classmates “How old are the students in my class in years?” will result in less variability than asking “How old are the students in my class in months?”  

Students might want to know about the fitness of the students at their school. Specifically, they want to know about the exercise habits of the
students. So rather than asking "Do you exercise?" they should ask about the amount of exercise the students at their school get per week. A statistical question for this study could be: “How many hours per week on average do students at Jefferson Middle School exercise?”

To collect this information, students might design a survey question that anticipates variability by providing a variety of possible anticipated responses that have numerical answers, such as: 3 hours per week, 4 hours per week, and so on. Be sure that students ask questions that have specific numerical answers.

The development of students’ understanding of statistical variability requires that they persevere in making sense of given data sets (MP.1). They also investigate how measures of center and variability model and summarize data (MP.4).

**Examples:**

- For each of the following, determine whether or not the question is a statistical question. Give a reason for your answer.
  - Who is my favorite movie star?
    *No, not answered by collecting data that vary.*
  - What are the favorite colors of 6th graders in my school?
    *Yes, colors will vary.*
  - How many years have students in my school’s band or orchestra played an instrument?
    *Yes, number of years will vary.*
  - How many brothers and sisters does my best friend have?
    *No, not answered by collecting data that vary.*

- Ronnie, a 6th grader, wanted to find out if he lived the farthest from school. Write a statistical question that would help Ronnie find the answer.
  *Sample Question: What is a typical distance (in miles) from home to school for students at my school?*
Write a statistical question that can be answered by collecting data from students in your class.

Answers will vary.

Change the following question to make it a statistical question: “How old is my math teacher?”

Sample Question: What is the typical age of teachers in my school?

Sixth graders should be able to examine the shape of data on a graph and determine which statistical question best fits the shape of the data.

Example:

- Which question best fits the dot plot below?
  - How many glasses of milk does each member of our class drink a day?
  - How many letters are in the first name of each class member?
  - How many text messages does each class member send in a day?
  - How many library books are in each class member’s desk?

Solution: #3 because the given numbers on the number line are not reasonable for glasses of milk, letters in a first name, or library books in a desk.

A. Develop understanding of statistical variability.

Understand that a set of data collected to answer a statistical question has a distribution that can be described by its center, spread, and overall shape.

Explanation:

In 1st grade students ask and answer questions about the data points in a graph. A study of line plots is provided in grades 2 through 5. Picture graphs and bar graphs are studied in grades 2 and 3. Therefore, 6th grade is a foundational year for dot plots, histograms and box plots.
6.MP.2. Reason abstractly and quantitatively.
6.MP.5. Use appropriate tools strategically.
6.MP.7. Look for and make use of structure.

The distribution is the arrangement of the values of a data set. Distribution can be described using center and spread. Data collected can be represented on graphs, which will show the shape of the distribution of the data. Students examine the distribution of a data set and discuss the center, spread and overall shape with dot plots and histograms.

The two dot plots show the 6-trait writing scores for a group of students on two different traits, organization and ideas. The center, spread and overall shape can be used to compare the data sets. Students consider the context in which the data were collected and identify clusters, peaks, gaps, and symmetry. Showing the two graphs vertically rather than side by side helps students make comparisons. For example, students would be able to see from the display of the two graphs that the ideas scores are generally higher than the organization scores. One observation students might make is that the scores for organization are clustered around a score of 3 whereas the scores for ideas are clustered around a score of 5.
Examples:

A teacher asks a question, “How many hours of TV do you watch in a week?” A graphic display of the data is above. What general observations can be made about the data?

Solution:
- The majority of students watch between 10 and 20 hours of TV
- One student watches almost double the number of hours that other students watch
- The minimum number of hours watched was 0 and the maximum was 38.
- The center of the data is around 15.
- 48 students took the survey.

Explanation:

In order to display numerical data in dot plots, histograms or box plots, students need to make decisions and perform calculations. Students are expected to display data graphically in a format appropriate for that data set as well as reading data from graphs generated by other students or contained in reference materials. Students can use applets to create data displays. Examples of applets include the Histogram Tool on NCTM’s Illuminations.

Histogram Tool --
http://illuminations.nctm.org/ActivityDetail.aspx?ID=78

Big Ideas:
Section 9.1, 10.2, 10.3
Dot plots are simple plots on a number line where each dot represents a piece of data in the data set. Dot plots are suitable for small to moderate size data sets and are useful for highlighting the distribution of the data including clusters, gaps, and outliers.

In most real data sets, there is a large amount of data and many numbers will be unique. A graph (such as a dot plot) that shows how many ones, how many twos, etc. would not be meaningful; however, a histogram can be used instead. Students organize the data into convenient ranges and use these intervals to generate a frequency table and histogram. Note that changing the size of the range changes the appearance of the graph and the conclusions you may draw from it.

Examples:

- Nineteen students completed a writing sample that was scored using the six traits rubric. The scores for the trait of organization were
  
  0, 1, 2, 2, 3, 3, 3, 3, 3, 4, 4, 4, 4, 5, 5, 5, 6, 6

  Create a data display. What are some observations that can be made from the data display?

  Solution:

  ![Dot Plot Example]

  - Most of the students scored between 3 and 4 (the data is clustered to the right)
  - The lowest score was a 0 and the highest was a 6
  - 3 was the score received by the most students

- Grade 6 students were collecting data for a math class project.
They decided they would survey the other two grade 6 classes to determine how many DVDs each student owns. A total of 48 students were surveyed. The data are shown in the table below in no specific order. Create a data display. What are some observations that can be made from the data display?

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</tbody>
</table>

**Solution:** A histogram using 5 ranges (0-9, 10-19, ...30-39) to organize the data is displayed below.

Most of the students have between 10 and 19 DVDs as indicated by the peak on the graph. The data is pulled to the right since only a few students own more than 30 DVDs.

<table>
<thead>
<tr>
<th></th>
<th>SP</th>
<th>5b</th>
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</thead>
<tbody>
<tr>
<td>B. Summarize and describe distributions.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Summarize numerical data sets in relation to their context, such as by: b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement. 6.MP.2. Reason abstractly and quantitatively. 6.MP.3. Construct viable arguments and critique</td>
<td></td>
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</tr>
<tr>
<td>Explanation: In Topic A, students summarize numerical data by providing background information about the attribute being measured, methods and unit of measurement, the context of data collection activities, the number of observations, and summary statistics. Summary statistics in Topic A include approximated center, spread, and variability including extreme values (minimum and maximum). Students record the number of observations. Using histograms, students determine the number of values between specified intervals. Reporting of the number of observations must consider the attribute of the data sets,</td>
<td></td>
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<tr>
<td>Eureka Math: M6 Lesson 1-5</td>
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<tr>
<td>Big Ideas: Section 9.1</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
the reasoning of others.

6.MP.5. Use appropriate tools strategically.
6.MP.7. Look for and make use of structure.

including units (when applicable).

Example:

- “What is the typical number of pets owned by Ms. Miller’s 6th grade students?” The dot plot below represents the data collected.

![Dot Plot]

- What was measured or counted by the statistical question? *Number of pets owned by each student*
- How was the attribute measured or counted? *Class survey*
- What unit of measurement is being used to describe the data? *Number of pets per student*
### 6th Grade Math Fourth Quarter

**Unit 6: Statistics**  
**Topic B: Analyzing and Summarizing Data**

In Topic B, students study quantitative ways to summarize numerical data sets in relation to their context and to the shape of the distribution. The mean and mean absolute deviation (MAD) are used for data distributions that are approximately symmetric, and the median and interquartile range (IQR) are used for distributions that are skewed. Students apply their experience in writing, reading, and evaluating expressions in which letters stand for numbers (6.EE.A.2) as they learn to compute and interpret two pairs of statistical measures for center and spread (6.SP.A.5). Students study mean as a measure of center and mean absolute deviation as a measure of variability. Students learn that these measures are preferred when the shape of the distribution is roughly symmetric. Then, students study median as a measure of center and interquartile range as a measure of variability. Students learn that these measures are preferred when the shape of the distribution is skewed. Students develop the idea that a measure of center provides a summary of all its values in a single number, while a measure of variation describes how values vary, also with a single number (6.SP.A.3). Students synthesize what they have learned as they connect the graphical, verbal, and numerical summaries to each other within situational contexts, culminating with a major project (6.SP.B.4, 6.SP.B.5). Students implement the four-step investigative process with their projects by stating their statistical questions, explaining the plan they used to collect data, analyzing data numerically and with graphs, and interpreting their results as related to their questions.

#### Big Idea:
- Statistical questions and the answers account for variability in the data.
- The distribution of a data set is described by its center, spread, and overall shape.
- Measures of center for a numerical set of data are summaries of the values using a single number.
- Measures of variability describe the variation of the values in the data set using a single number.

#### Essential Questions:
- How do we analyze and interpret data sets?
- When is one data display better than another?
- How do mathematicians choose to display data in strategic ways?
- When is one statistical measure better than another?
- What is the difference between symmetric and skewed distributions?

#### Vocabulary
- box plots, dot plots, histograms, mean, median, interquartile range, measures of center, measures of variability, data, deviation from the mean, Mean Absolute Deviation (M.A.D.), quartiles, lower quartile (1st quartile or Q1), upper quartile (3rd quartile or Q3), symmetrical, skewed, summary statistics, outlier

<table>
<thead>
<tr>
<th>Grade</th>
<th>Domain</th>
<th>Standard</th>
<th>AZ College and Career Readiness Standards</th>
<th>Explanations &amp; Examples</th>
<th>Resources</th>
</tr>
</thead>
</table>
| 6     | SP     | 2        | A. Develop understanding of statistical variability.  
Understand that a set of data collected to answer a statistical question has a distribution that can be | Explanation:  
The distribution is the arrangement of the values of a data set. Distribution can be described using center (median or mean), and spread. Data collected can be represented on graphs, which will show | Eureka Math:  
M6 Lesson 6-22  
Big Ideas:  
Section 9.2-9.5, 10.2-10.4 |
described by its center, spread, and overall shape.

6.MP.2. Reason abstractly and quantitatively.
6.MP.5. Use appropriate tools strategically.
6.MP.7. Look for and make use of structure.

the shape of the distribution of the data. Students examine the
distribution of a data set and discuss the center, spread and overall
shape with dot plots, histograms and box plots.

The two dot plots show the 6-trait writing scores for a group of
students on two different traits, organization and ideas. The center,
spread and overall shape can be used to compare the data sets.

Students consider the context in which the data were collected and
identify clusters, peaks, gaps, and symmetry. Showing the two graphs
vertically rather than side by side helps students make comparisons.

For example, students would be able to see from the display of the two
graphs that the ideas scores are generally higher than the organization
scores. One observation students might make is that the scores for
organization are clustered around a score of 3 whereas the scores for
ideas are clustered around a score of 5.

6-Trait Writing Rubric
Scores for Organization

0 1 2 3 4 5 6

6-Trait Writing Rubric
Scores for Ideas

0 1 2 3 4 5 6

Examples:
- Consider the data shown in the dot plot of the six trait scores
  for organization for a group of students.
  - How many students are represented in the data set?
<table>
<thead>
<tr>
<th>6</th>
<th>SP</th>
<th>3</th>
<th>A. Develop understanding of statistical variability.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.</td>
</tr>
</tbody>
</table>

**Explanation:**

Data sets contain many numerical values that can be summarized by one number such as a measure of center. The measure of center gives a numerical value to represent the center of the data (ie. midpoint of an ordered list or the balancing point). Another characteristic of a data set is the variability (or spread) of the values. Measures of variability are used to describe this characteristic.

When using measures of center (mean, median, and mode) and range, students are describing a data set in a single number. The range provides a single number that describes how the values vary across the data set. The range can also be expressed by stating the minimum and maximum values.

The development of students’ understanding of statistical variability requires that they persevere in making sense of given data sets (MP.1). They also investigate how measures of center and variability model and summarize data (MP.4).

**Example:**

- Consider the data shown in the dot plot of the six trait scores.

**Solution:**

The values range from 0 – 6. There is a peak at 3. The median is 3, which means 50% of the scores are greater than or equal to 3 and 50% are less than or equal to 3. The mean is 3.68. If all students scored the same, the score would be 3.68.

**NOTE:** Mode as a measure of center and range as a measure of variability are not addressed in the CCSS and as such are not a focus of instruction. These concepts can be introduced during instruction as needed.
6. SP 4  

| 6 | SP | 4 |  

### B. Summarize and describe distributions.

Display numerical data in plots on a number line, including dot plots, histograms, and box plots.

- **6.MP.2.** Reason abstractly and quantitatively.
- **6.MP.4.** Model with mathematics.
- **6.MP.5.** Use appropriate tools strategically.
- **6.MP.6.** Attend to precision.

In order to display numerical data in dot plots, histograms or box plots, students need to make decisions and perform calculations. Students are expected to display data graphically in a format appropriate for that data set as well as reading data from graphs generated by others students or contained in reference materials. Students can use applets to create data displays. Examples of applets include the Box Plot Tool and Histogram Tool on NCTM’s Illuminations.

Box Plot Tool - [http://illuminations.nctm.org/ActivityDetail.aspx?ID=77](http://illuminations.nctm.org/ActivityDetail.aspx?ID=77)

---

**Eureka Math:**

- **M6 Lesson 6-22**

**Big Ideas:**

- Section 10.2, 10.3, 10.4
6.MP.7. Look for and make use of structure.

Histogram Tool --
http://illuminations.nctm.org/ActivityDetail.aspx?ID=78

Dot plots are simple plots on a number line where each dot represents a piece of data in the data set. Dot plots are suitable for small to moderate size data sets and are useful for highlighting the distribution of the data including clusters, gaps, and outliers.

In most real data sets, there is a large amount of data and many numbers will be unique. A graph (such as a dot plot) that shows how many ones, how many twos, etc. would not be meaningful; however, a histogram can be used. Students organize the data into convenient ranges and use these intervals to generate a frequency table and histogram. Note that changing the size of the range changes the appearance of the graph and the conclusions you may draw from it.

Box plots are another useful way to display data and are plotted horizontally or vertically on a number line. Box plots are generated from the five number summaries of a data set consisting of the minimum, maximum, median, and two quartile values. Students can readily compare two sets of data if they are displayed with side by side box plots on the same scale. Box plots display the degree of spread of the data and the skewness of the data.

Examples:

- Nineteen students completed a writing sample that was scored using the six traits rubric. The scores for the trait of organization were 0, 1, 2, 2, 3, 3, 3, 3, 3, 3, 4, 4, 4, 4, 5, 5, 5, 6, 6. Create a data display. What are some observations that can be made from the data display?

  ![](image)

- Grade 6 students were collecting data for a math class project. They decided they would survey the other two grade 6 classes...
to determine how many DVDs each student owns. A total of 48 students were surveyed. The data are shown in the table below in no specific order. Create a data display. What are some observations that can be made from the data display?

<table>
<thead>
<tr>
<th>Number of DVDs Students Own</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-9</td>
</tr>
<tr>
<td>11</td>
</tr>
<tr>
<td>10</td>
</tr>
<tr>
<td>22</td>
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<tr>
<td></td>
</tr>
</tbody>
</table>

Solution: A histogram using 5 ranges (0-9, 10-19, ...30-39) to organize the data is displayed below.

Most of the students have between 10 and 19 DVDs as indicated by the peak on the graph. The data is pulled to the right since only a few students own more than 30 DVDs.

• Ms. Wheeler asked each student in her class to write their age in months on a sticky note. The 28 students in the class brought their sticky note to the front of the room and posted them in order on the white board. The data set is listed below in order from least to greatest. Create a data display. What are some observations that can be made from the data display?
Solution:

**Five number summary**

Minimum – 130 months

Quartile 1 (Q1) – \((132 + 133) \div 2 = 132.5\) months

Median (Q2) – 139 months

Quartile 3 (Q3) – \((142 + 143) \div 2 = 142.5\) months

Maximum – 150 months

<table>
<thead>
<tr>
<th>130</th>
<th>130</th>
<th>131</th>
<th>131</th>
<th>132</th>
<th>132</th>
<th>133</th>
<th>134</th>
<th>136</th>
</tr>
</thead>
<tbody>
<tr>
<td>137</td>
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<td>144</td>
<td>145</td>
<td>147</td>
<td>149</td>
<td>150</td>
<td></td>
</tr>
</tbody>
</table>

This box plot shows that

- \(\frac{1}{4}\) of the students in the class are from 130 to 132.5 months old
- \(\frac{1}{4}\) of the students in the class are from 142.5 months to 150 months old
- \(\frac{1}{2}\) of the class are from 132.5 to 142.5 months old
- The median class age is 139 months.

**B. Summarize and describe distributions.**

Summarize numerical data sets in relation to their context, such as by:

a. Reporting the number of observations.

b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement.

**Explanation:**

Students summarize numerical data by providing background information about the attribute being measured, methods and unit of measurement, the context of data collection activities, the number of observations, and summary statistics. Summary statistics include quantitative measures of center, spread, and variability including extreme values (minimum and maximum), mean, median, mode, range, quartiles, interquartile ranges, and mean absolute deviation.
c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.

d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.

6.MP.2. Reason abstractly and quantitatively.
6.MP.3. Construct viable arguments and critique the reasoning of others.
6.MP.5. Use appropriate tools strategically.
6.MP.7. Look for and make use of structure.

Students record the number of observations. Using histograms, students determine the number of values between specified intervals. Given a box plot and the total number of data values, students identify the number of data points that are represented by the box. Reporting of the number of observations must consider the attribute of the data sets, including units (when applicable).

Mean Absolute Deviation (MAD) describes the variability of the data set by determining the absolute deviation (the distance) of each data piece from the mean and then finding the average of these deviations.

Both the interquartile range and the Mean Absolute Deviation are represented by a single numerical value. Higher values represent a greater variability in the data.

Students understand how the measures of center and measures of variability are represented by graphical displays.

Students describe the context of the data, using the shape of the data and are able to use this information to determine an appropriate measure of center and measure of variability. The measure of center that a student chooses to describe a data set will depend upon the shape of the data distribution and context of data collection. The mode is the value in the data set that occurs most frequently. The mode is the least frequently used as a measure of center because data sets may not have a mode, may have more than one mode, or the mode may not be descriptive of the data set. The mean is a very common measure of center computed by adding all the numbers in the set and dividing by the number of values. The mean can be affected greatly by a few data points that are very low or very high. In this case, the median or middle value of the data set might be more descriptive. In data sets that are symmetrically distributed, the mean and median will be very close to the same. In data sets that are skewed, the mean and median will be different, with the median frequently providing a better overall description of the data set.

Summarizing and describing distributions requires that students use appropriate statistical measures strategically (MP.5) and attend to precision in finding and applying those measures (MP.6). Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered requires
that students construct and critique arguments (MP.3). Students look for and apply regularity in repeated reasoning when they explore how adding or removing data points from a data set affects the measures of center and variability (MP.8).

### Measures of Center
Given a set of data values, students summarize the measure of center with the median or mean. The median is the value in the middle of a ordered list of data. This value means that 50% of the data is greater than or equal to it and that 50% of the data is less than or equal to it.

#### Measures of Center

**Given a set of data values, students summarize the measures of center with the median or mean.** The median is the value in the middle of a ordered list of data. This value means that 50% of the data is greater than or equal to it and that 50% of the data is less than or equal to it.

**The mean** is the arithmetic average; the sum of the values in a data set divided by how many values there are in the data set. The mean measures center in the sense that it is the value that each data point would take on if the total of the data values were redistributed equally, and also in the sense that it is a balance point.

Students develop these understandings of what the mean represents by redistributing data sets to be level or fair (equal distribution) and by observing that the total distance of the data values above the mean is equal to the total distance of the data values below the mean (balancing point).

Students use the concept of mean to solve problems. Given a data set represented in a frequency table, students calculate the mean.

**Students find a missing value in a data set to produce a specific average.**

**Understanding the Mean**

Students develop understanding of what the mean represents by redistributing data sets to be level or fair. The leveling process can be connected to and used to develop understanding of the computation of the mean.

For example, students could generate a data set by measuring the number of jumping jacks they can perform in 5 seconds, the length of their foot to the nearest inch, or the number of letters in their names. It is best if the data generated for this activity are 5 to 10 data points which are whole numbers between 1 and 10 that are easy to model with counters or...
Examples:

- Students generate a data set by drawing eight student names at random from the popsicle stick cup. The number of letters in each of the names is used to create the data set. If the names drawn were Carol, Mike, Maria, Luis, Monique, Sierra, John, and Karen there would be 3 names with 4 letters each, 3 names with 5 letters each, 1 name with 6 letters and 1 name with 7 letters.

“If all of the students had the same number of letters in their name, how many letters would each person have?”

Solution:

This data set could be represented with stacking cubes.

Students can model the mean by “leveling” the stacks or distributing the blocks so the stacks are “fair”.

One block from the stack of six and two blocks from the stack of 7 can be moved down to the stacks of 4 and then all the stacks have five blocks. If all students had the same number of letters in their name, they would have five letters. The mean number of letters in a name in this data set is 5.

If it was not possible to make the stacks exactly even, students...
could begin to consider what part of the extra blocks each stack would have.

- Susan has four 20-point projects for math class. Susan’s scores on the first 3 projects are shown below:
  - Project 1: 18
  - Project 2: 15
  - Project 3: 16
  - Project 4: ??

  What does she need to make on Project 4 so that the average for the four projects is 17? Explain your reasoning.

  **Solution:**
  One possible solution to is calculate the total number of points needed (17 x 4 or 68) to have an average of 17. She has earned 49 points on the first 3 projects, which means she needs to earn 19 points on Project 4 (68 – 49 = 19).

**Measures of Variability**
Measures of variability/variation can be described using the interquartile range or the Mean Absolute Deviation. The interquartile range (IQR) describes the variability between the middle 50% of a data set. It is found by subtracting the lower quartile from the upper quartile. It represents the length of the box in a box plot and is not affected by outliers.

Students find the IQR from a data set by finding the upper and lower quartiles and taking the difference or from reading a box plot.

- **Understanding Mean Absolute Deviation**
  The use of mean absolute deviation in 6th grade is mainly exploratory. The intent is to build a deeper understanding of variability. Students would understand the mean distance between the pieces of data and the mean of the data set expresses the spread of the data set. Students can see that the larger the mean distance, the greater the variability.
Comparisons can be made between different data sets.

In the popsicle stick example above, the names drawn were Carol, Mike, Maria, Luis, Monique, Sierra, John, and Karen. There were 3 names with 4 letters each, 3 names with 5 letters each, 1 name with 6 letters and 1 name with 7 letters. This data can be represented on a dot plot. The mean of the data set is 5.

```
   o o
   o o
 o o o o o
   3 4 5 6 7 8
```

To find the mean absolute deviation, students examine each of the data points and its difference from the mean. This analysis can be represented on the dot plot itself or in a table. Each of the names with 4 letters has one fewer letter than the mean, each of the names with 5 letters has zero difference in letters as compared to the mean, each of the names with 6 letters has one more letter than the mean, and each of the names with 7 letters has two more letters than the mean. The absolute deviations are the absolute value of each difference.

```
    -1  0
Deviations from the mean    -1  0  +1  +2
    3  4  5  6  7  8
```
The mean of the absolute deviations is found by summing the absolute deviations and dividing by the number of data points. In this case, the mean absolute deviation would be \(6 \div 8 = 0.75\). The mean absolute deviation is a small number, indicating that there is little variability in the data set.

**Examples:**

- Consider a different data set also containing 8 names. If the names were Sue, Joe, Jim, Amy, Sabrina, Monique, Timothy, and Adelita. Summarize the data set and its variability. How does this compare to the first data set?

**Solution:**
The mean of this data set is still 3.
The mean deviation of this data set is $16 \div 8$ or 2. Although the mean is the same, there is much more variability in this data set.

- The following data set represents the size of 9 families:
  
  $3, 2, 4, 2, 9, 8, 2, 11, 4$

  What is the MAD for this data set?

  \[ 3 + 3 + 3 + 7 + 7 + 7 + 7 \]
  
  \[ \frac{8}{8} = 5 \]

  Solution:
  
  The mean is 5. The MAD is the average variability of the data set. To find the MAD:
  
  o Find the deviation from the mean.
  
  o Find the absolute deviation for each of the values from step 1
  
  o Find the average of these absolute deviations.

  The table below shows these calculations:

<table>
<thead>
<tr>
<th>Name</th>
<th>Number of letters in a name</th>
<th>Deviation from the Mean</th>
<th>Absolute Deviation from the Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sue</td>
<td>3</td>
<td>-2</td>
<td>2</td>
</tr>
<tr>
<td>Joe</td>
<td>3</td>
<td>-2</td>
<td>2</td>
</tr>
<tr>
<td>Jim</td>
<td>3</td>
<td>-2</td>
<td>2</td>
</tr>
<tr>
<td>Amy</td>
<td>3</td>
<td>-2</td>
<td>2</td>
</tr>
<tr>
<td>Sabrina</td>
<td>7</td>
<td>+2</td>
<td>2</td>
</tr>
<tr>
<td>Timothy</td>
<td>7</td>
<td>+2</td>
<td>2</td>
</tr>
<tr>
<td>Adelita</td>
<td>7</td>
<td>+2</td>
<td>2</td>
</tr>
<tr>
<td>Monique</td>
<td>7</td>
<td>+2</td>
<td>2</td>
</tr>
<tr>
<td>Total</td>
<td>40</td>
<td>0</td>
<td>16</td>
</tr>
</tbody>
</table>

The mean deviation of this data set is $16 \div 8$ or 2. Although the mean is the same, there is much more variability in this data set.
This value indicates that on average family size varies 2.89 from the mean of 5.

- **Understanding Medians and Quartiles**
  Students can also summarize and describe the center and variability in data sets using the median and a five number summary consisting of the minimum, quartiles, and maximum as seen in the box plot example in 6.SP.4. The median is the middle number of the data set with half the number below the median and half the numbers above the median. The quartiles partition the data set into four parts by dividing each of the halves of the data set into half again. Quartile 1 (Q1 or the lower quartile) is the middle value of the lower half of the data set and quartile 3 (Q3 or the upper quartile) is the middle value of the upper half of the data set. The median can also be referred to as quartile 2 (Q2). The range of the data is the difference between the minimum and maximum values. The interquartile range of the data is the difference between the lower and upper quartiles (Q3 – Q1). The interquartile range is a measure of the dispersion or spread of the data set: a small value indicates values that are clustered near the median whereas a larger value indicates values that are more distributed.
Examples:

• Consider the popsicle stick example again. Recall that the names drawn were Carol, Mike, Maria, Luis, Monique, Sierra, John, and Karen. The data set can be represented in a numerical list. Find the median and quartiles of the data.

Solution:
To find the median and quartile, the values are placed in order from least to greatest.

\[
5 \quad 4 \quad 5 \quad 4 \quad 7 \quad 6 \quad 4 \quad 5 \quad 4 \quad 4 \quad 5 \quad 5 \quad 6 \quad 7
\]

The middle value in the ordered data set is the median. If there are an even numbers of values, the median is the mean of the middle two values. In this case, the median would be 5 because 5 is the average of the 4\textsuperscript{th} and 5\textsuperscript{th} values which are both 5.

Students find quartile 1 (Q1) by examining the lower half of the data. Again there are 4 values which is an even number of values. Q1 would be the average of the 2\textsuperscript{nd} and 3\textsuperscript{rd} value in the data set or 4. Students find quartile 3 (Q3) by examining the upper half of the data. Q3 would be the average of the 6\textsuperscript{th} and 7\textsuperscript{th} value in the data set or 5.5.

The mean of the data set was 5 and the median is also 5, showing that the values are probably clustered close to the mean. The interquartile range is 1.5 (5.5 – 4). The interquartile range is small, showing little variability in the data.

\[
\begin{align*}
\text{Q1} &= 4 \\
\text{Q3} &= 5.5 \\
\text{Median} &= 5
\end{align*}
\]

• What is the IQR of the data below:
Solution:
The first quartile is 132.5; the third quartile is 142.5. The IQR is 10 (142.5 – 132.5). This value indicates that the values of the middle 50% of the data vary by 10.